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ABSTRACT

The books in this series provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. They aim to guide the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The main objective of the activities in this book is for the student to be able to build any design of the pattern. The activities allow students to use rods of graduated lengths to build and analyze the given designs, then determine the relationship between the number of the design and the number of rods necessary to build the design. Exploration and discovery of individual solution strategies are encouraged. The book is laid out in an easy-to-use format with 50 pattern pages (divided into three sections), an appendix (including teaching guide and a glossary), and an answer key. Within the five sections included, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. (JRH)

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RODS

Building Algebraic Thinking with Progressive Patterns

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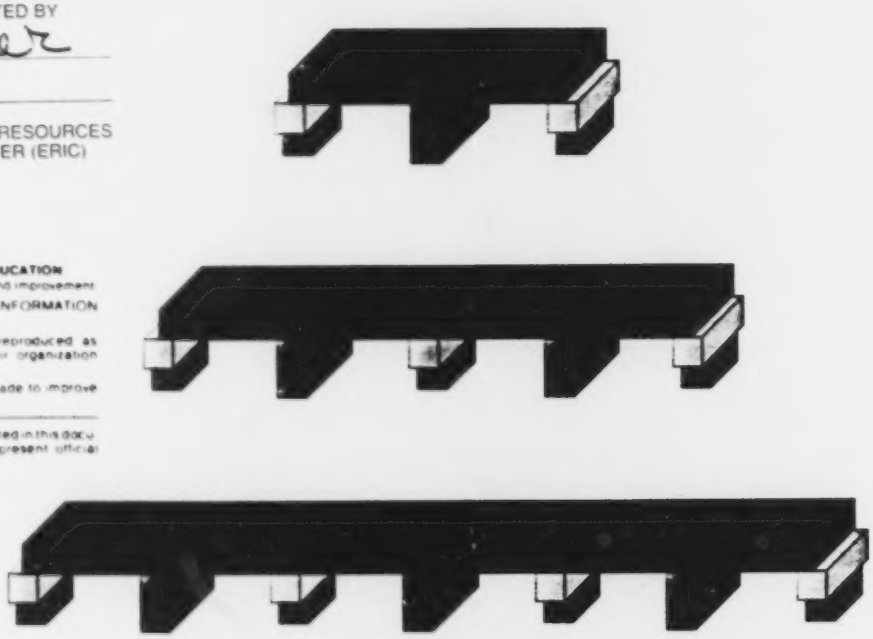

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RODS

Building Algebraic Thinking with Progressive Patterns

Bob Willcutt

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ABOUT THE PROGRAM

Goals

The books *Building Algebraic Thinking with Progressive Patterns* (*CUBES, RODS,* and *PATTERN BLOCKS*) provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. The program guides the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The activities engage creativity and provide positive early experiences with a variable, helping students make a smooth transition into the study of algebra. *RODS* focuses on the use of Cuisenaire-type rods; *CUBES,* on cubic blocks (such as Unifix cubes); and *PATTERN BLOCKS*, on pattern blocks (varied shapes). The books are self-contained and do not need to be used in any special order.

Building Algebraic Thinking is a discovery-based approach, using a questioning strategy to provide problem solving experiences as recommended by the National Council of Teachers of Mathematics (N.C.T.M.). By following the given strategy and building patterns with manipulative materials, students develop the skills needed to

- substitute for a variable in an equation
- write generalized equations using a variable
- find creative solutions based on geometric reasoning
- combine like variables and numbers
- factor
- reverse process

Throughout the book, exploration and discovery of individual solution strategies are encouraged. Ultimate success is attained when students can create and solve designs of their own.

Objective/Methods

On each activity page, the objective is for the student to be able to build any design of the pattern. To do this, the student uses rods of graduated lengths to build and analyze the given designs and then determines the relationship between the number of the design and the number of rods necessary to build the design. As the pattern progresses from the first design (design number 1) to the 4th and 5th designs, students will notice that the number of rods required to build each successive design increases in a predictable way. This results in a "generalization," a general equation, using the variable n, that applies to all designs of the pattern. The design number is substituted for n to get the number of rods needed to build the design. For example, on page 1, the student finds that multiplying the design number by two gives the total number of rods needed to build that design of the pattern (design 3 requires $3 \times 2 = 6$ rods). Substituting n for the design number, the student can give a generalization of $n \times 2 =$ number of rods needed. To find the number of rods needed for building design 12, substitute 12 for n to get $12 \times 2 = 24$ blocks.

The manipulative activities program can be used with individuals, small groups, and whole classes. Questions on the activity, or pattern, pages are let-

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tered sequentially, but the suggested questions do not have to be answered in the given order. There is no "right" way for students to come to the correct generalization, and they should be encouraged to find their own methods.

Based on the columns on the Number of Rods page, students may recognize a pattern of regularly increasing numbers. If the numbers always grow by 2, the student puts that number before n to get $2n$ for the total number of rods. This "constant difference" method is a systematic and efficient tool for solving even the most challenging generalizations.

On the other hand, "visual" learners may best see the generalization by relating the physical, or spatial, arrangements of rods to the total number of rods. If there are 2 rods in each of 5 rows for design 5, there are $2n$ rods in all. If the generalization is not readily seen in the given arrangement, students may want to rearrange rods to make the generalization more obvious.

Some students find the generalization by the constant difference method and then explain how it is verified by the rod arrangement. Enlisting the creative imagination (What does this design remind you of?) can enhance the learning process. Whatever methods are used, it is important that students share ideas and see that there can be a variety of creative ways to find generalizations.

Using the Book

Building Algebraic Thinking is laid out in an easy-to-use format with 50 pattern pages (divided into 3 sections), an appendix (including 3 reproducible Number of Rods pages, a teaching guide, and a glossary), and an answer key. Each pattern includes a progression (numbered 1 through n) of designs, each consistent with the overall pattern. All pattern problems involve a variable (n) and require rods of two or three different lengths from 1 to 10 units. Problems become increasingly more complex in each of the book's three sections; section 3 involves computation of lengths of rods as well as numbers of rods. Answers are found in the answer key at the end of the book.

Each student (or cooperative group) should have one Number of Rods page for each pattern page and enough rods to build as many designs as necessary; the number of designs built may vary with the students. (See the appendix for reproducible Number of Rods pages.) Following the illustrated pattern as a model, students use the rods to build the designs; the designs can be used to generate the number data for the Number of Rods page. For each design built, students must record the numbers of rods needed. This task reinforces the importance of collecting correct data, as do many science-based activities, because one wrong entry will break the emerging pattern!

The questions on each pattern page help direct students in analyzing relationships between the numbers of the designs and the numbers of rods to determine and express a generalized equation using the variable n. To foster students' experience with a variable, you might ask, "What would happen if I decided to use n to stand for any number? The n could be 10 or 580 or whatever we want it to be. What will we put in the Total column if we use n for the design number?" (See chart at right.) The answer will be $2n$, or $n + n$.

Spans (Design)	Dark Green	Light Green	Total
1	1	1	2
2	2	2	4
3	3	3	6
4	4	4	8
5	5	5	10
⋮	⋮	⋮	⋮
10	10	10	20
⋮	⋮	⋮	⋮
20	20	20	40
⋮	⋮	⋮	⋮
n			?

Students apply the generalization to determine the number of rods required for

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any design of the pattern. Both forward processing (design 2 takes how many rods?) and reverse processing (which design will take 20 rods?) are required. Even if the students do not yet have the tools for formal algebraic operations, by exploring with concrete materials, students form a foundation for understanding later algebraic operations.

Within the three sections of the book, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. Some of the final exercises (exploring quadratic solutions) may seem very difficult, but students may use calculators to aid exploration (trial and error is encouraged). Though some students will no longer actually need to build the patterns after a few pages, others may continue to need the concrete experience into the final section.

Because *Building Algebraic Thinking* is a discovery-based program, it is recommended that teachers use a flexible approach, gearing the program to students' abilities. For those who feel the need for specific guidance, however, a questioning guide is provided in the appendix.

Specific information about each of the three pattern sections can be found in the chart that follows.

CONTENTS OF MANIPULATIVE ACTIVITIES PAGES

	Manipulatives	Design Formats	Solution Types	Skills/Features	Suggestions
SECTION 1 (Pages 1–15)	Rods of unit lengths 1–10; two specified types of rods per pattern	Bridges with growing numbers of spans	Linear	Basic multiplication Reverse processing; Building confidence (mental arithmetic, class discussion, exploration)	•Allow students to actually build the designs (this is crucial) • Let students themselves discover the relationship between constant difference and coefficient of "n" • Extensions: have students (1) create own patterns and (2) graph data on an "xy" graph
SECTION 2 (Pages 16–30)	Rods of unit lengths 1–10; three specified types of rods per pattern	Bridges with growing numbers of spans	Linear	Combining like terms Factoring as a foundation Reverse processing Constant difference method Seeing a connection between the constant difference and the coefficient is not crucial at this stage	• Give the first 3 designs of a creative pattern and have students build next (some may come up with a divergent, though still consistent, design); discuss the question, When is a pattern clearly established?
SECTION 3 (Pages 31–50)	Rods of any specified length per pattern	Steplike structures	Pages 31–42: Linear Pages 43–50: Quadratic (becoming very challenging)	Combining like terms Reverse processing Combining creativity and mathematics skills Using a variable to substitute for length Many students may not need the building experience; some may need an intermediate step to the abstract	• For a good spatial activity similar to those on standardized tests, show the first three designs of a pattern and have students create the next one • Encourage analysis of both physical arrangement and constant difference • Extensions: have students (1) create own patterns and (2) for a specific pattern, compute total volume for each of several rod lengths

Manipulatives

It is very important that students see how the manipulative designs generate the associated numerical pattern; therefore, they must actually build the designs before going on to the number work. Not all students will continue the manipulative work throughout the book, but for those who need concrete experiences (and even adults do!), building and extending the designs is crucial and will provide the primary benefit. Students with special needs get a lot of satisfaction from con-

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tinually extending the patterns. Students can always return to the numerical work at a later date.

Each student should have access to enough rods to build the designs. Commercial materials are available, but the important thing is to provide students with rods of definite, graduated lengths. The designs used in this book are based on Cuisenaire rods, which conform to the characteristics listed at right.

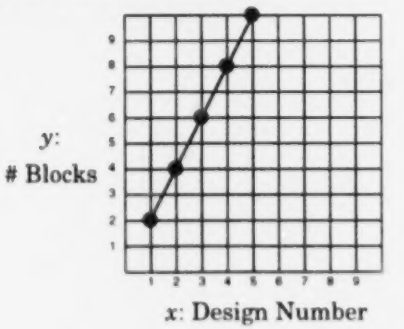
Rod Color	Unit Length
White	1
Red	2
Light Green	3
Purple	4
Yellow	5
Dark Green	6
Black	7
Brown	8
Blue	9
Orange	10

Extension Activities

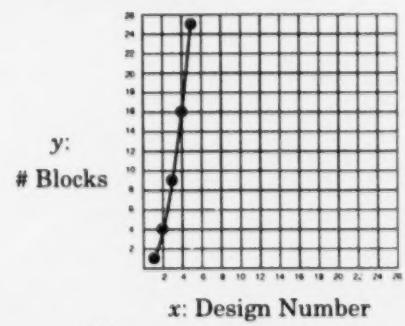
The activities below can provide a valuable extension to the regular manipulative activities.

- Creating patterns: Have students make their own designs adhering to the criteria for a pattern. Remind them that progressive designs must be consistent with the overall pattern; patterns must be developed to the point where, based on the given designs, there is only one subsequent design that can logically follow. Have students share their own patterns with other students or their families and challenge them to solve for the generalization. A copy of the pattern will make an appropriate entry in a student's learning portfolio. Also, each student could contribute one pattern page to a class pattern book.
- Variations: Discuss structural problems, architectural designs, and costs; calculate the cubic units of concrete needed for each span of a design.
- Graphing the equations: Give students graph paper and have them label and mark the x-axis with design numbers and the y-axis with the numbers of rods used. Have them plot the number of rods for each design and join the points. Guide them in recognizing the relationship of the line (straight or curved) to the type of solution required for the generalized equation (linear or quadratic). See graphing examples below.

Linear Function Graph



Quadratic Function Graph



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PATTERN 1

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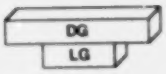

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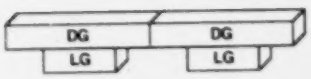

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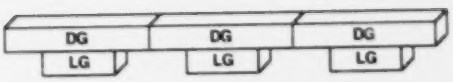

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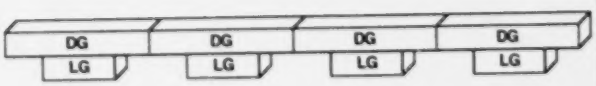

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A. The bridge with 30 spans will need ____ dark green rods (DG) and ____ light green rods (LG).

B. When we use 46 dark green rods, what is the total number of rods we will use?

____

C. As you build the pattern designs and fill in the Total column on the Number of Rods page, notice how the numbers change from one design to the next. What must be done to each number to get the next number?

_____

_____

_____

What kind of numbers make up the pattern (odd, even, counting numbers)?

_____

_____

D. When the total number of rods is 98, how many rods will be light green? ____

E. Will we ever use exactly 129 rods? ____ Why?

_____

_____

_____

F. The bridge with 279 spans will use a total of ____ rods.

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PATTERN 2

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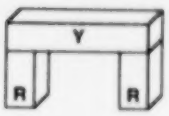

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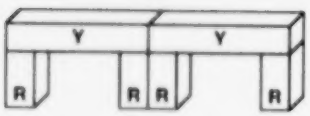

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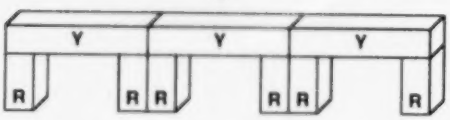

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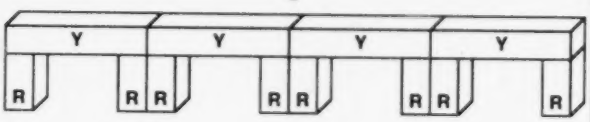

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A. The bridge with 26 spans will need _____ yellow rods (Y) and _____ red rods (R).

B. When we use 48 red rods, how many yellow rods will we need? _____

C. As you build the pattern and fill in the blanks on the Number of Rods page, notice how the numbers change from one design to the next. What kinds of numbers make up the number pattern in each column?

Yellow _____

Red _____

Total _____

D. Will we ever use 87 yellow rods? _____ Why?

_____

_____

_____

Will we ever use 87 red rods? _____ Why?

_____

_____

_____

Will we ever have a total of 120 rods? _____ Why?

_____

_____

_____

E. When the total number of rods is 141, _____ will be red rods.

F. The bridge with 189 spans will use _____ red rods and have a total of _____ rods.

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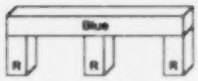
PATTERN 3

1



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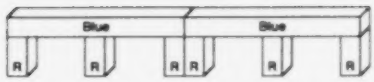

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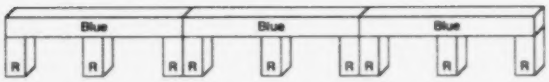

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A. The bridge with 33 spans will need ____ blue rods (Blue) and _____ red rods (R).

B. As you build the pattern and fill in the blanks on the Number of Rods page, notice how the numbers change from one design to the next. What kinds of numbers make up the number pattern in each column?

Blue _____

Red _____

Total _____

C. Tell whether we will ever use each of the following numbers of rods and explain why:

78 blue _____

_____

78 red _____

_____

78 total _____

_____

D. By what amounts do the numbers in each column grow?

Blue ____ Red ____ Total ____

How does the growing number in each column relate to the generalization for *n* in that column?

_____

_____

E. When the total number of rods is 252, _____ will be red rods.

F. The bridge with 159 spans will use _____ red rods and have a total of _____ rods.

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PATTERN 4

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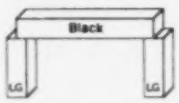

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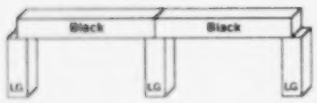

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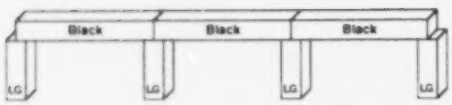

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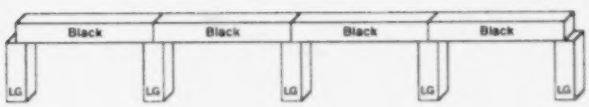

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A. The bridge with 38 spans will need _____ black rods (Black) and _____ light green rods (LG).

B. As you fill in the blanks on the Number of Rods page, by how much do the column numbers grow?

Black _____ Light green _____

Total _____

How does the growing number in each column relate to the generalization for *n* in that column?

_____

_____

C. Will the total number of rods ever be an even number? _____ Why?

_____

_____

_____

D. When the total number of rods is 119, _____ will be black rods.

E. The bridge with 187 spans will use _____ rods in all.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

_____

_____

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PATTERN 5

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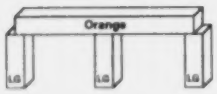

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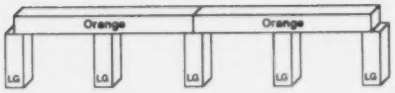

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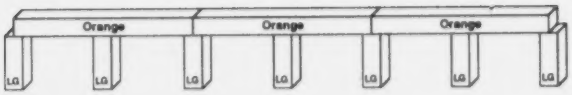

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A. The bridge with 35 spans will need _____ orange rods (Orange) and _____ light green rods (LG).

B. As you fill in the blanks on the Number of Rods page, by how much do the column numbers grow?

Orange _____ Light green _____

Total _____

How does the growing number in each column relate to the generalization for *n* in that column?

_____

_____

_____

C. Will we ever use exactly 97 light green rods? _____ Why?

_____

_____

Will we ever use a total of exactly 97 rods? _____ Why?

_____

_____

D. When the total number of rods is 142, _____ will be light green rods.

E. The bridge with 87 spans will need _____ light green rods and have a total of _____ rods.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

_____

_____

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PATTERN 6

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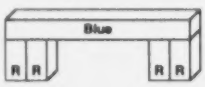

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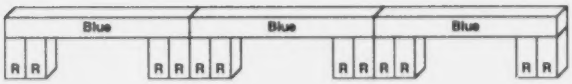

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A. The bridge with 39 spans will need _____ blue rods and _____ red rods (R).

B. Look at the number patterns created in the Red column and in the Total column on the Number of Rods page. What kinds of numbers are they?

Red _____

Total _____

C. When we use 152 red rods, the total number of rods will be _____.

D. Will we ever use 236 red rods? _____ Why?

_____

Will we ever use 236 rods in all? _____ Why?

_____

E. By how much do the numbers grow in the columns on the Number of Rods page?

Blue _____ Red _____ Total _____

How does the growing number (constant difference) in each column relate to the generalization for *n* in that column?

_____

_____

F. The bridge with 79 spans will need _____ red rods and will have a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization for the total number of rods.

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14

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PATTERN 7

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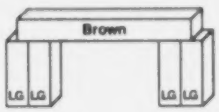

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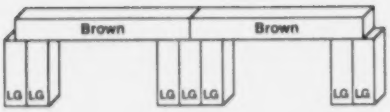

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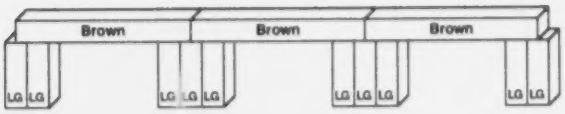

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A. The bridge with 34 spans will need _____ brown rods (Brown) and _____ light green rods (LG).

B. Will we ever use an even total number of rods? _____ Why?

_____

_____

C. When we use 118 light green rods, we will need _____ brown rods.

D. By how much do the column numbers grow on the Number of Rods page?

Brown _____ Light Green _____

Total _____

How does the growing number (constant difference) in each column relate to the generalization for n in that column?

_____

_____

E. Will we ever use 189 light green rods? _____ Why?

_____

_____

Will we ever use 189 total rods? _____ Why?

_____

_____

F. The bridge with 87 spans will use _____ light green rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 8

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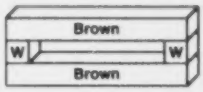

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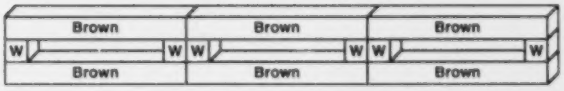

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A. The bridge with 39 spans will need _____ brown rods and _____ white rods (W).

B. Describe the number patterns created in the columns on the Number of Rods page. What kinds of numbers are these?

Brown _____

White _____

Total _____

C. When we use 114 white rods, we will need a total of _____ rods.

D. Will we ever have a total of 155 rods? _____ Why?

_____

_____

E. By how much do the column numbers on the Number of Rods page grow?

Brown _____ White _____

Total _____

How does the growing number (constant difference) in each column relate to the generalization for *n* in that column?

_____

_____

F. The bridge with 219 spans will use _____ brown rods and a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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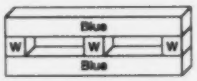
PATTERN 9

1



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A. The bridge with 43 spans will need _____ blue rods and _____ white rods (W).

B. Describe the number patterns created in the columns on the Number of Rods page. What kinds of numbers are these?

Blue _____

White _____

Total _____

C. When we use 96 white rods, we will need _____ blue rods.

D. Tell whether we will ever use each of the following numbers of rods and why:

126 blue _____

_____

126 white _____

_____

126 total _____

_____

E. By how much do the column numbers grow on the Number of Rods page?

Blue _____ White _____ Total _____

How does each constant difference relate to that column's generalization?

_____

F. The bridge with 229 spans will use _____ blue rods and _____ white rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

_____

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PATTERN 10

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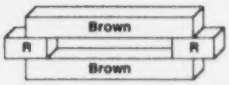

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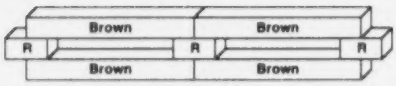

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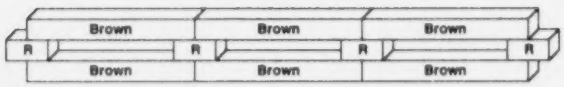

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A. The bridge with 47 spans will need _____ red rods (R) and _____ brown rods.

B. Tell whether we will ever use each of the following numbers of rods and explain why:

118 red _____

_____

_____

118 brown _____

_____

_____

118 total _____

_____

_____

C. When we use a total of 172 rods, _____ will be red rods and _____ will be brown rods.

D. By how much do the column numbers on the Number of Rods page grow?

Red _____ Brown _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

E. The bridge with 237 spans will need _____ brown rods and a total of _____ rods.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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18

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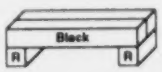
PATTERN 11

1



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A. The bridge with 45 spans will need ____ black rods and ____ red rods (R).

B. Look at the number patterns created in the columns on the Number of Rods page. What kinds of numbers are these?

Black _____

Red _____

Total _____

C. When we use a total of 228 rods, how many will be red rods? ____

D. Will we ever use 214 red rods? ____ Why?

_____

Will we ever have a total of 214 rods? ____ Why?

_____

E. By how much do the column numbers increase on the Number of Rods page?

Black ____ Red ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. The bridge with 276 spans will need ____ red rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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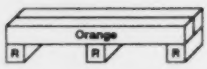
PATTERN 12

1



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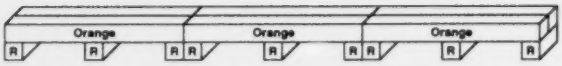

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A. The bridge with 49 spans will need _____ orange rods and _____ red rods (R).

B. Look at the number patterns created in the columns on the Number of Rods page. What are the numbers called?

Orange _____

Red _____

Total _____

C. When we use a total of 255 rods, we use _____ orange rods and _____ red rods.

D. Tell whether we will ever use each of the following numbers of rods and why:

135 orange _____

_____

135 red _____

_____

135 total _____

_____

E. By how much do the column numbers grow on the Number of Rods page?

Orange _____ Red _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

F. The bridge with 291 spans will need _____ orange rods and _____ red rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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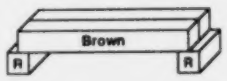
PATTERN 13

1



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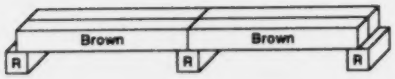

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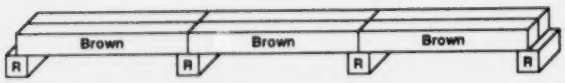

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A. The bridge with 47 spans will need _____ red rods (R) and _____ brown rods.

B. Tell whether we will ever use each of the following numbers of rods and why:

202 red _____

_____

202 brown _____

_____

202 total _____

_____

C. When we use a total of 250 rods, _____ will be red rods and _____ will be brown rods.

D. By what amounts do the column numbers grow on the Number of Rods page?

Red _____ Brown _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

E. The bridge with 249 spans will need _____ red rods and _____ brown rods.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

_____

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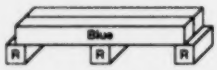
PATTERN 14

1



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A. The bridge with 39 spans will need _____ blue rods and _____ red rods (R).

B. Tell whether we will ever use each of the following numbers of rods and why:

187 blue _____

_____

187 red _____

_____

187 total _____

_____

C. When we use a total of 265 rods, _____ will be blue rods and _____ will be red rods.

D. By what amounts do the columns on the Number of Rods page grow?

Blue _____ Red _____ Total _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

E. The number of blue rods is always an even number while the number of red rods is always an odd number. Explain why their sum is always an odd number.

_____

_____

F. The bridge with 263 spans will need _____ blue rods and _____ red rods for a total of _____ rods.

22

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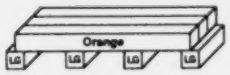
PATTERN 15

1



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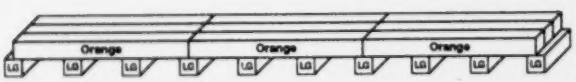

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A. The bridge with 37 spans will need _____ orange and _____ light green (LG) rods.

B. Tell whether we will ever use each of the following numbers of rods and why:

162 orange _____

_____

183 light green _____

_____

203 total _____

_____

C. When we use a total of 295 rods, _____ will be orange rods and _____ will be light green rods.

D. By how much do the column numbers on the Number of Rods page grow?

Orange _____ Light green _____

Total _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

E. Why is the total number of rods never an even number?

_____

_____

F. The bridge with 229 spans will need _____ orange rods and _____ light green rods for a total of _____ rods.

23

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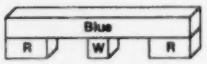
PATTERN 16

1



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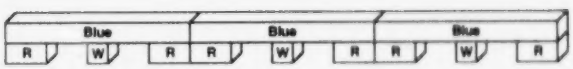

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A. The bridge with 47 spans will need _____ blue rods, _____ white rods (W), and _____ red rods (R).

B. Describe the number patterns created in the columns on the Number of Rods page. What kinds of numbers are these?

Blue _____

White _____

Red _____

Total _____

C. When we use a total of 236 rods, _____ will be blue rods.

D. Will we ever use a total of 350 rods? _____
Why? _____

_____

E. By how much do the column numbers grow on the Number of Rods page (what are the constant differences)?

Blue _____ White _____ Red _____

Total _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

F. The bridge with 198 spans will need a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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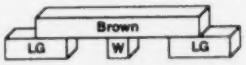
PATTERN 17

1



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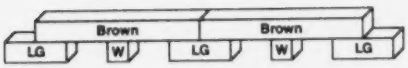

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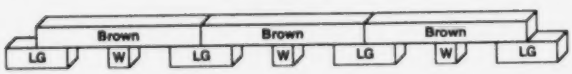

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A. The bridge with 43 spans will need _____ brown rods, _____ white rods (W), and _____ light green rods (LG).

B. When we use 57 light green rods, we will need a total of _____ rods.

C. By what amount do the numbers increase in the Total column on the Number of Rods page (what is the constant difference)?

_____

_____

_____

How does the constant difference in the Total column relate to the generalization for *n* in that column?

_____

_____

_____

D. When we use a total of 172 rods, _____ will be white rods and _____ will be light green rods.

E. Will we ever use a total of 204 rods? _____ Why? _____

_____

F. The bridge with 207 spans will need a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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PATTERN 18

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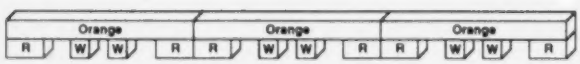

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A. The bridge with 45 spans will need _____ orange rods, _____ red rods (R), and _____ white rods (W).

B. Look at the number patterns created in the columns on the Number of Rods page. What kinds of numbers are they?

Orange _____

Red _____

White _____

Total _____

C. When we use 78 white rods, we will need a total of _____ rods.

D. By how much do the column numbers grow on the Number of Rods page?

Orange _____ Red _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

E. Will we ever use a total of 204 rods? _____ Why? _____

F. When we use a total of 295 rods, _____ will be orange rods, _____ will be red rods, and _____ will be white rods.

G. The bridge with 179 spans will need a total of _____ rods.

H. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

_____

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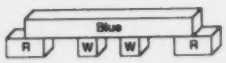
PATTERN 19

1



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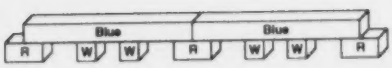

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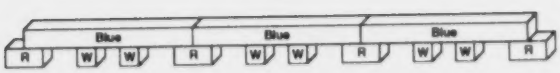

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A. The bridge with 49 spans will need ____ blue rods, ____ red rods (R), and ____ white rods (W).

B. When we use 96 white rods, we will need a total of ____ rods.

C. By how much do the column numbers grow on the Number of Rods page?

Blue ____ Red ____

White ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

D. Will we ever use a total of 292 rods? ____
Why? _____

_____

Will we ever use a total of 293 rods? ____
Why? _____

_____

E. When we use a total of 357 rods, ____ will be red rods, and ____ will be white.

F. The bridge with 189 spans will need a total of ____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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PATTERN 20

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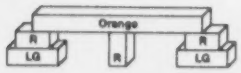

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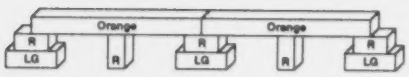

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A. The bridge with 47 spans will need _____ light green rods (LG), _____ red rods (R), and a total of _____ rods.

B. When we use 57 light green rods, we will need _____ red rods.

C. By what amounts do the numbers grow in the columns on the Number of Rods page?

Orange _____ Light green _____

Red _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

D. Will we ever use a total of 276 rods? _____ Why?

_____

_____

Will we ever use 276 red rods? _____ Why?

_____

_____

E. When we use a total of 350 rods, _____ will be orange, _____ will be light green, and _____ will be red.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 21

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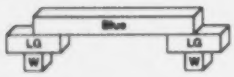

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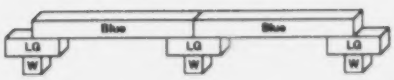

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A. The bridge with 53 spans will need _____ blue rods, _____ light green rods (LG), and _____ white rods (W) for a total of _____ rods.

B. When we use 49 white rods, we will need a total of _____ rods.

C. By what amounts do the numbers grow in the columns on the Number of Rods page?

Blue _____ Light green _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

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D. Will we ever use a total of 197 rods? _____ Why?

_____

_____

E. When we use a total of 239 rods, _____ will be white rods.

F. The bridge with 165 spans will need _____ light green rods and a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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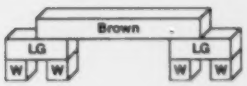
PATTERN 22

1



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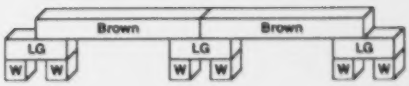

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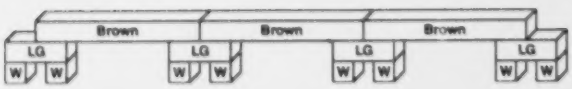

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A. The bridge with 46 spans will need _____ brown rods, _____ light green rods (LG), and _____ white rods (W) for a total of _____ rods.

B. When we use 78 white rods, we will need a total of _____ rods.

C. By what amounts do the numbers grow in the columns on the Number of Rods page?

Brown _____ Light green _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

D. Will we ever use a total of 253 rods? _____ Why?

_____

_____

_____

E. When we use a total of 319 rods, _____ will be white rods.

F. The bridge with 183 spans will need _____ white rods and a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 23

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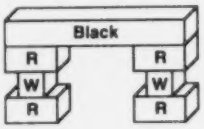

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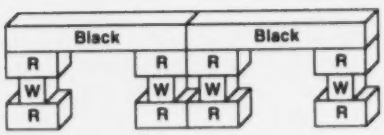

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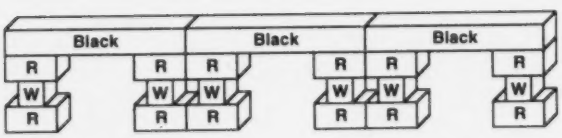

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A. The bridge with 48 spans will need ____ black rods, ____ red rods (R), and ____ white rods (W), for a total of ____ rods.

B. Look at the number patterns created in the columns on the Number of Rods page. What are the numbers called?

Black _____

Red _____

White _____

Total _____

C. When we use 156 red rods, we will need a total of ____ rods.

D. Will we ever use 186 white rods? ____ Why?

_____

Will we ever use 186 red rods? ____ Why?

_____

E. By what amounts do the numbers grow in the columns on the Number of Rods page?

Black _____ Red _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When we use a total of 371 rods, ____ will be red and ____ will be white.

G. The bridge with 197 spans will need ____ black rods, ____ red rods, ____ white rods, and a total of ____ rods.

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PATTERN 24

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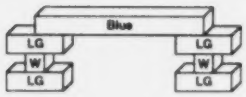

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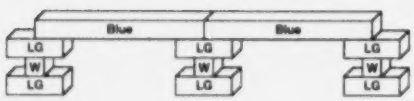

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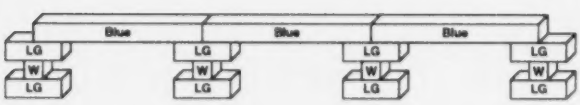

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A. The bridge with 51 spans will need _____ blue rods, _____ light green rods (LG), and _____ white rods (W), for a total of _____ rods.

B. When we use 68 light green rods, we will need a total of _____ rods.

C. By how much do the numbers grow in the columns on the Number of Rods page?

Blue _____ White _____

Light green _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

D. Will we ever use 235 light green rods? _____ Why?

_____

_____

Will we ever use a total of 235 rods? _____ Why?

_____

_____

E. When we use a total of 271 rods, _____ will be white rods.

F. The bridge with 195 spans will need _____ light green rods and a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 25

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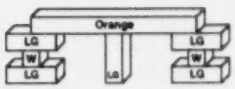

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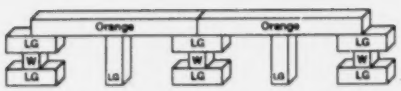

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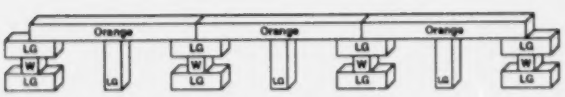

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A. The bridge with 49 spans will need _____ orange rods, _____ light green rods (LG), and _____ white rods (W), for a total of _____ rods.

B. When we use 113 light green rods, we will need a total of _____ rods.

C. By how much do the numbers grow in the columns on the Number of Rods page?

Orange _____ White _____

Light green _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

D. Will we ever use 213 light green rods? _____ Why?

_____

_____

Will we ever have a total of 213 rods? _____ Why?

_____

_____

E. When we use a total of 288 rods, _____ will be white rods and _____ will be light green rods.

F. The bridge with 167 spans will need a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 26

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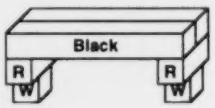

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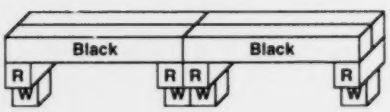

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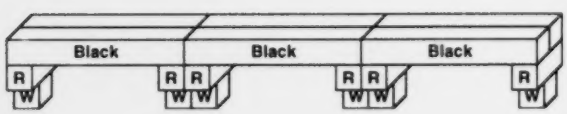

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A. The bridge with 57 spans will need _____ black rods, _____ red rods (R), and _____ white rods (W) for a total of _____ rods.

B. Look at the number patterns created in the columns on the Number of Rods page. What kinds of numbers are they?

Black _____

Red _____

White _____

Total _____

C. When we use 86 red rods, we will need a total of _____ rods.

D. By what amounts do the numbers grow in the columns?

Black _____ Red _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

E. Will we ever use 292 white rods? _____
Why? _____

Will we ever use 292 rods in all? _____
Why? _____

F. When we use a total of 354 rods, _____ will be black rods.

G. The bridge with 259 spans will need _____ red rods and a total of _____ rods.

H. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 27

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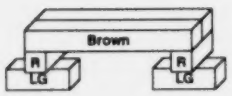

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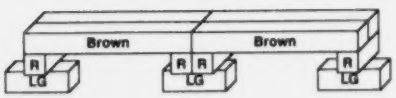

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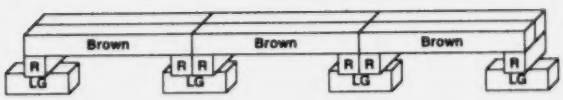

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A. The bridge with 46 spans will need ____ brown rods, ____ light green rods (LG), and ____ red rods (R) for a total of ____ rods.

B. When we use 74 red rods, we will need a total of ____ rods.

C. By how much do the numbers grow in the columns on the Number of Rods page?

Brown ____ Red ____

Light green ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

D. Tell whether we will ever use each of the following numbers of blocks and why:

121 light green _____

_____

121 red _____

_____

121 total _____

_____

E. When we use a total of 436 rods, ____ will be brown rods.

F. The bridge with 219 spans will need ____ brown rods, ____ light green rods, and a total of ____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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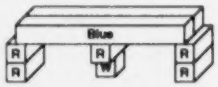
PATTERN 28

1



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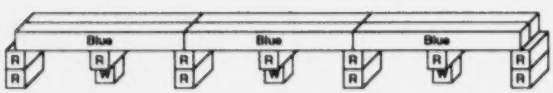

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A. The bridge with 51 spans will need _____ blue rods, _____ red rods (R), and _____ white rods (W), for a total of _____ rods.

B. When we use 89 red rods, we will need a total of _____ rods.

C. By how much do the numbers grow in each column on the Number of Rods page?

Blue _____ Red _____

White _____ Total _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

D. Will we ever use 119 red rods? _____ Why?

_____

_____

Will we ever use a total of 119 rods? _____ Why?

_____

_____

E. When we use a total of 296 rods, _____ will be blue rods.

F. The bridge with 187 spans will need _____ blue rods, _____ red rods, _____ and white rods for a total of _____ rods.

G. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

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PATTERN 29

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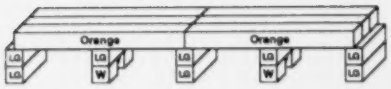

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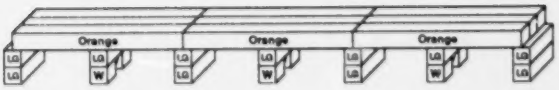

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A. The bridge with 54 spans will need _____ white rods (W), _____ orange rods, and _____ light green rods (LG), for a total of _____ rods.

B. When we use 84 orange rods, we will need a total of _____ rods.

C. By how much do the numbers in the columns grow on the Number of Rods page?

White _____ Orange _____

Light green _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

D. Will we ever use 130 orange rods? _____ Why?

_____

_____

Will we ever use a total of 538 rods? _____ Why?

_____

_____

E. When we use a total of 394 rods, _____ will be light green rods.

F. The bridge with 139 spans will need _____ white rods, _____ orange rods, and _____ light green rods for a total of _____ rods.

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PATTERN 30

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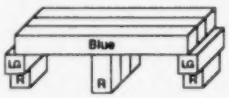

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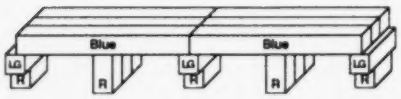

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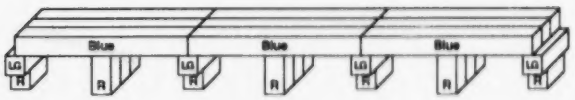

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A. The bridge with 59 spans will need _____ light green rods (LG), _____ blue rods, and _____ red rods (R) for a total of _____ rods.

B. When we use 96 blue rods, what will be the total number of rods? _____

C. By how much do the numbers grow in each column on the Number of Rods page?

Light green _____ Blue _____

Red _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

D. Will we ever use 269 red rods? _____ Why?

_____

_____

Will we ever use a total of 572 rods? _____ Why?

_____

_____

E. When we use a total of 634 rods, _____ will be red rods.

F. Explain why the sum of the generalizations of each color column equals the generalization in the Total column.

_____

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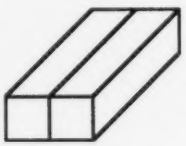
PATTERN 31

1



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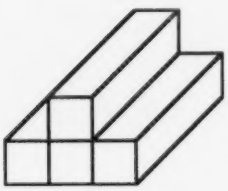

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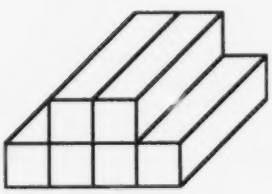

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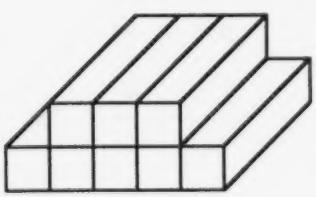

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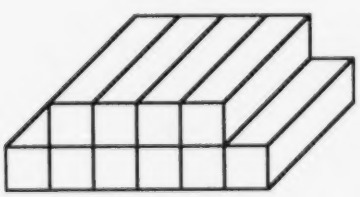

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Build the pattern three times, using yellow, then purple, then dark green rods.

A. How many rods will the 59th design require? _____

B. As you compute the number of rods and the total lengths required for each color used, fill in the columns on the Number of Rods page. By how much do the numbers grow in each column?

Number of rods _____ Yellow _____

Purple _____ Dark green _____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

_____

C. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods required for the nth design.

Black _____

Brown _____

Rods of length l _____

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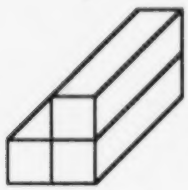
PATTERN 32

1



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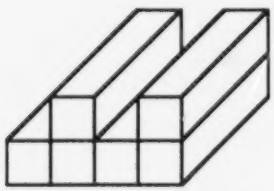

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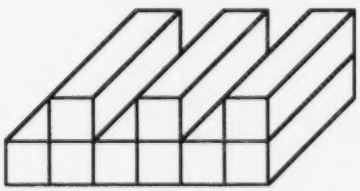

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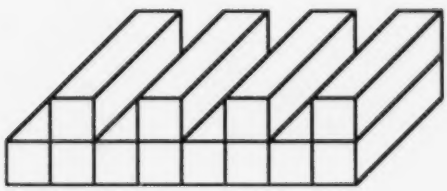

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Build the pattern three times, using yellow, then purple, then dark green rods.

A. The 63rd design will need _____ rods.

B. As you compute the number of rods and the total lengths required for each color used, fill in the columns on the Number of Rods page. By how much do the numbers grow in each column?

Number of rods _____ Yellow _____

Purple _____ Dark Green _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

C. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the *n*th design.

Black _____

Brown _____

Rods of length *l* _____

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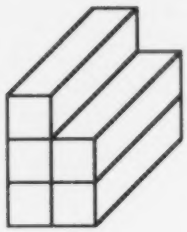
PATTERN 33

1



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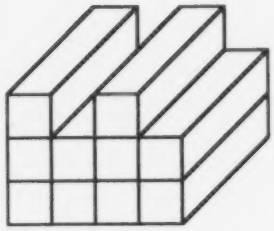

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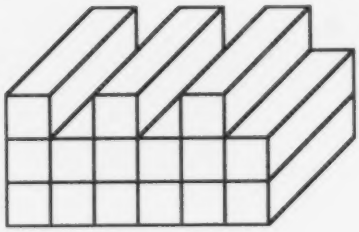

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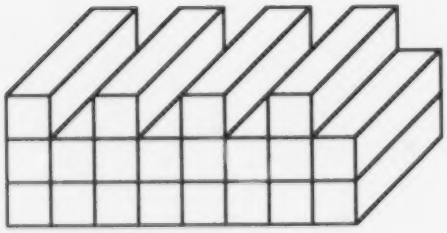

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Build the pattern three times, using yellow, then purple, then dark green rods.

A. How many rods will you need for the 66th design? _____

B. Will we ever use exactly 142 rods? _____ Why?

_____

_____

C. As you fill in the columns on the Number of Rods page, by how much do the numbers grow in each?

Number of rods _____ Yellow _____

Purple _____ Dark green _____

How does the constant differece in each column relate to the generalization for n in that column?

_____

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Black _____

Brown _____

Rods of length l _____

41

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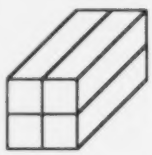
PATTERN 34

1



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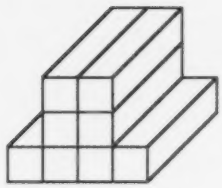

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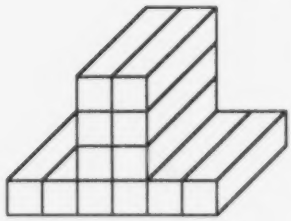

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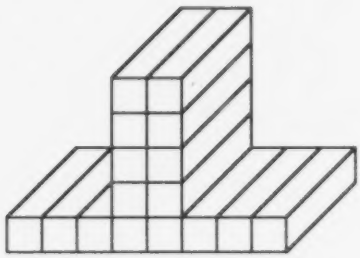

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Build the pattern three times, using yellow, then purple, then dark green rods.

A. The 73rd design will need _____ rods.

B. Will we ever use exactly 346 rods? _____ Why?

_____

_____

_____

C. As you fill in the columns on the Number of Rods page, by how much do the numbers grow in each?

Number of rods _____ Yellow _____

Purple _____ Dark Green _____

How does the constant difference in each column relate to the generalization for *n* in each column?

_____

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the *n*th design.

Black _____

Brown _____

Rods of length *l* _____

42

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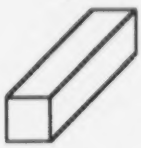
PATTERN 35

1



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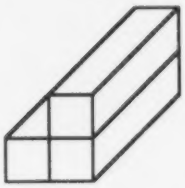

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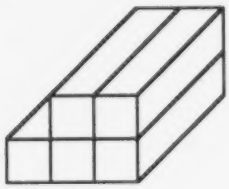

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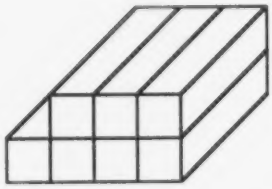

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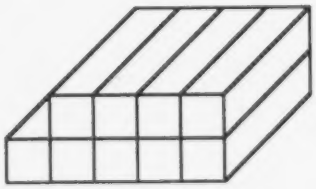

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Build the pattern three times, using yellow, then purple, then dark green rods.

A. How many rods will be required for the 71st design? _____

B. Will we ever use exactly 200 rods? _____ Why?

_____

_____

_____

C. When giving the generalization for the total length using purple rods, one student wrote $8n - 4$, while another student wrote $4(2n - 1)$. Why are both answers correct?

_____

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Black _____

Brown _____

Rods of length l _____

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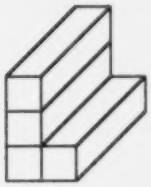
PATTERN 36

1

2



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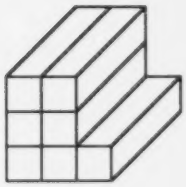

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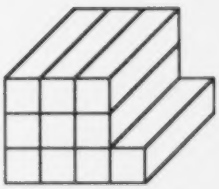

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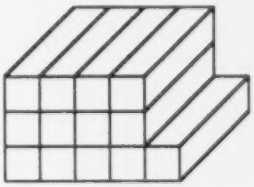

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Build the pattern three times, using yellow, then light green, then purple rods.

A. The 68th design will need _____ rods.

B. When giving the generalization for the total length of the light green rods, one student wrote $3(3n - 2)$, while another student wrote $9n - 6$. Why are both answers correct?

_____

_____

_____

C. Explain one way in which the rod pattern arrangements show the generalization of $3n - 2$.

_____

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Dark green _____

Black _____

Rods of length l _____

41

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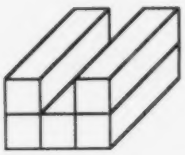
PATTERN 37

1



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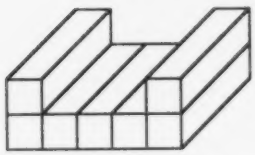

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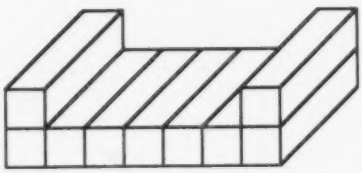

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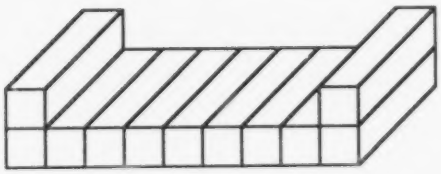

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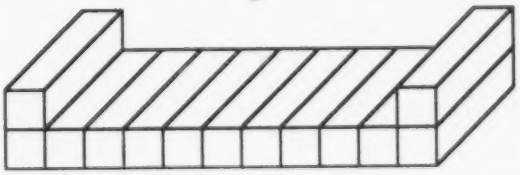

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Build the pattern three times, using yellow, then purple, then black rods.

A. The 74th design will need _____ rods.

B. Will we ever use exactly 121 rods? _____ Why?

_____

_____

_____

C. When giving the generalization for the total length of the black rods, one student wrote $14n + 21$, while another student wrote $7(2n + 3)$. Why are both answers correct?

_____

_____

_____

D. The exact number of rods needed for the _____ design will be 177.

E. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Light green _____

Dark green _____

Rods of length l _____

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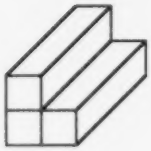
PATTERN 38

1

2



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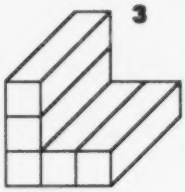

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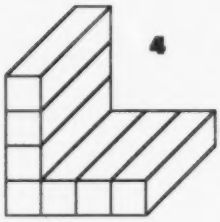

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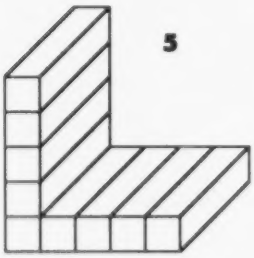

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Build the pattern three times, using yellow, then black, then blue rods.

A. How many rods will you need for the 69th design? _____

B. The numbers of rods used for the designs on this page ($2n - 1$; see Number of Rods page) are the same as those for the designs on page 35. Explain one way in which the rod patterns are the same.

_____

_____

_____

C. The exact number of rods needed for the _____ design will be 199.

D. When giving the generalization for the total length of yellow rods, one student wrote $5(2n - 1)$ while another student wrote $10n - 5$. Why are both answers correct?

_____

_____

_____

E. Give the generalization for the total length of rods when each rod has a length of l.

_____

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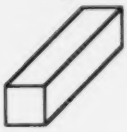
PATTERN 39

1



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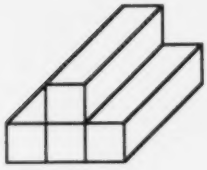

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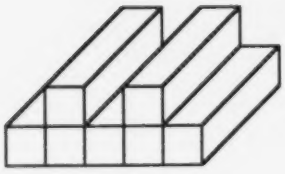

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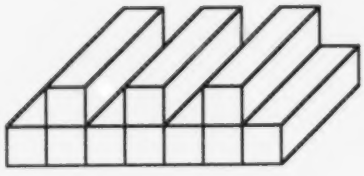

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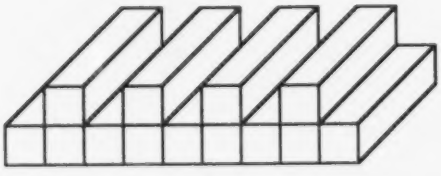

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n

Build the pattern three times, using black, then brown, then blue rods.

A. The 74th design will need _____ rods.

B. The numbers of rods needed for the designs on this page ($3n - 2$) are the same as the numbers of rods needed on page 36. Explain one way you could change the rod patterns on this page so that they will be the same as the rod patterns on page 36.

_____

_____

_____

C. Which design will use exactly 247 rods? _____

D. For each type of rod listed below, both of the given total lengths are correct. Explain why the expressions in each pair below are equivalent.

Black: $7(3n - 2)$ or $21n - 14$

_____

_____

Brown: $8(3n - 2)$ or $24n - 16$

_____

_____

Blue: $9(3n - 2)$ or $27n - 18$

_____

_____

E. Give the generalization for the total length of rods for rods of length l.

_____

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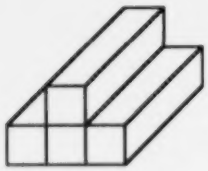
PATTERN 40

1



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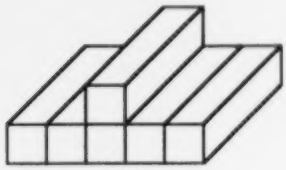

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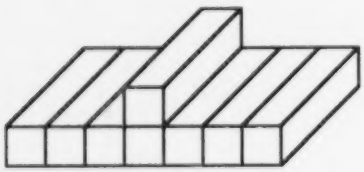

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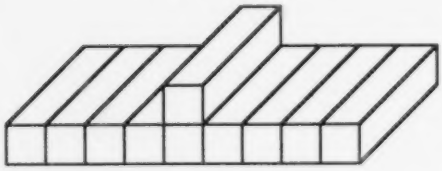

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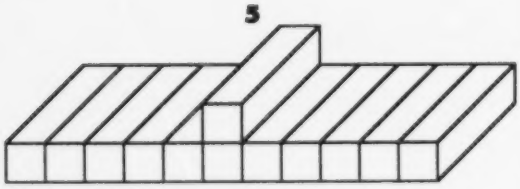

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n

Build the pattern three times, using purple, then dark green, then black rods.

A. How many rods will the 62nd design need? ____

B. The numbers of rods needed for this page ($2n + 2$) are nearly the same as the numbers of rods used on page 37 ($2n + 3$). Explain one way you could change the rod patterns on page 37 to be similar to the rod patterns on this page.

_____

_____

_____

C. Which design will need exactly 156 rods?

____

D. For each type of rod listed below, both of the given total lengths are correct. Explain why the expressions in each pair below are equivalent.

Purple: $4(2n + 2)$ or $8n + 8$

_____

_____

Dark green: $6(2n + 2)$ or $12n + 12$

_____

_____

Black: $7(2n + 2)$ or $14n + 14$

_____

_____

E. Give the generalization for the total length of rods for rods of length l.

_____

48

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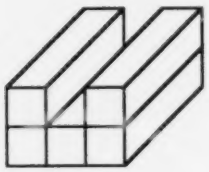
PATTERN 41

1



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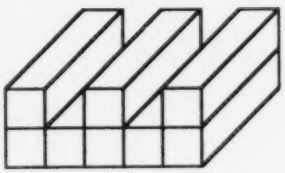

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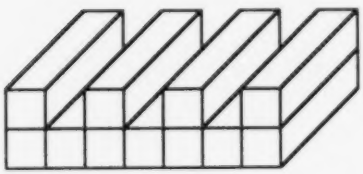

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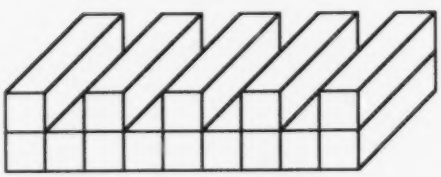

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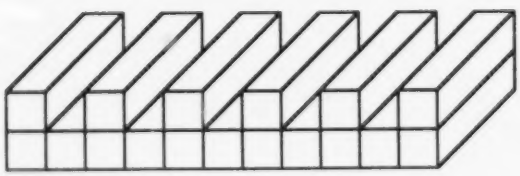

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n

Build the pattern three times, using dark green, then brown, then blue rods.

A. The 79th design will need _____ rods.

B. Which design will need exactly 275 rods? _____

C. The numbers of rods needed for this page ($3n + 2$) are nearly the same as the numbers of rods needed for page 32 ($3n$). Explain one way you could change the rod patterns on page 32 to be the same as the rod patterns on this page.

_____

_____

_____

D. For each type of rod listed below, both of the given total lengths are correct. Explain why the expressions in each pair below are equivalent.

Dark green: $6(3n + 2)$ or $18n + 12$

_____

_____

Brown: $8(3n + 2)$ or $24n + 16$

_____

_____

Blue: $9(3n + 2)$ or $27n + 18$

_____

_____

E. Give the generalization for the total length of rods for rods of length l.

_____

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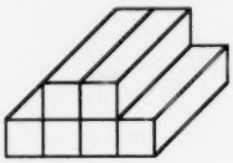
PATTERN 42

1



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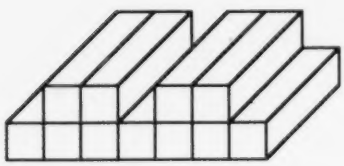

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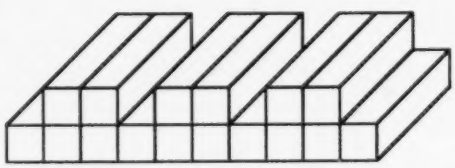

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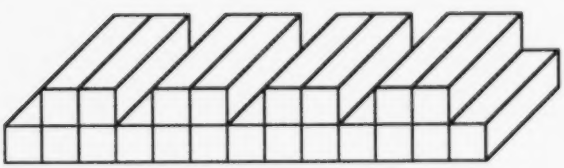

⋮

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n

Build the pattern three times, using light green, then yellow, then brown rods.

A. For the 66th design, how many rods will be needed? ____

B. The ____ design will need exactly 366 rods.

C. The numbers of rods needed for this page ($5n + 1$) are nearly the same as the numbers of rods needed for page 33 ($5n$). Explain one way you could change the rod patterns on this page to be similar to the rod patterns on page 33.

_____

_____

_____

D. For each type of rod listed below, both of the given total lengths are correct. Explain why the expressions in each pair below are equivalent.

Light green: $3(5n + 1)$ or $15n + 3$

_____

_____

Yellow: $5(5n + 1)$ or $25n + 5$

_____

_____

Brown: $8(5n + 1)$ or $40n + 8$

_____

_____

E. Give the generalization for the total length of rods for rods of length l.

_____

50

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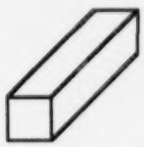
PATTERN 43

1



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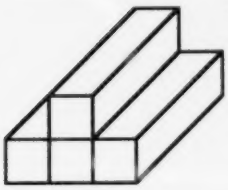

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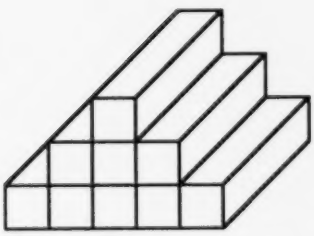

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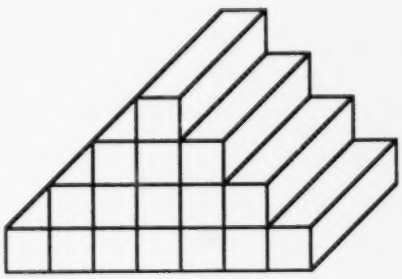

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n

Build the pattern three times, using light green, then purple, then yellow rods.

A. The 34th design will need _____ rods.

B. Which design requires exactly 1681 rods?
_____

C. By how much do the numbers grow in the columns on the Number of Rods page?

Number of rods _____

Light green _____

Purple_____

Yellow_____

When is there a constant difference?

_____

_____

_____

How much are the constant differences for the columns listed below?

Number of rods _____
Light green _____
Purple_____Yellow_____

How does the growing pattern relate to the generalization for n?

_____

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Dark green _____

Black _____

Rods of length l _____

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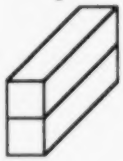
PATTERN 44

1



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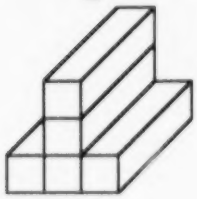

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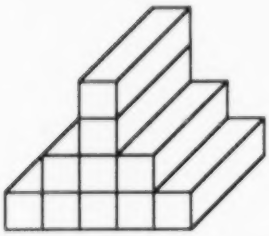

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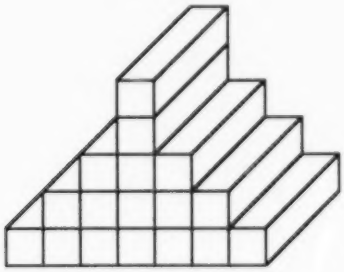

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10

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n

Build the pattern three times, using yellow, then dark green, then black rods.

A. The 37th design will need _____ rods.

B. Which design needs exactly 842 rods?

_____

C. By what amounts do the numbers grow in the columns on the Number of Rods page?

Number of Rods _____

Yellow _____

Dark green _____

Black _____

When is there a constant difference?

_____

_____

_____

How much are the constant differences for the colums listed below?

Number of Rods _____

Yellow _____

Dark green _____ Black _____

How do the growing patterns (constant differences) relate to the generalizations for n?

_____

_____

D. Assume the pattern is built with each of the following colors of rods. For each rod color, give the total length of rods for the nth design.

Brown rods _____

Blue rods _____

Rods of length l _____

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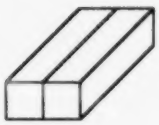
PATTERN 45

1



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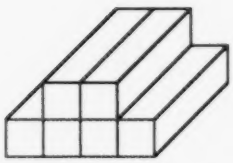

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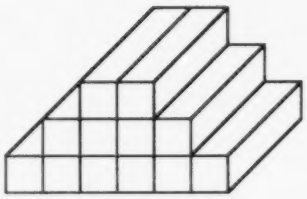

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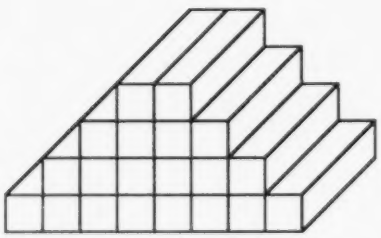

⋮

10

⋮

20

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n

Build the pattern three times, using purple, then yellow, then dark green rods.

A. How many rods will be needed for the 35th design? _____

B. For the _____ design, 1482 rods will be the exact number of rods required.

C. By how much do the numbers grow in the columns on the Number of Rods page?

Number of Rods _____

Purple _____

Yellow _____

Dark Green _____

When is there a constant difference?

_____

_____

How much are the constant differences for the colums listed below?

Number of Rods _____ Purple _____

Yellow _____ Dark green _____

How do the constant differences relate to the generalization for n?

_____

_____

D. The generalization for the number of rods can be expressed as either $n^2 + n$ or $n(n + 1)$. How can the rods be rearranged to clearly show that both answers are correct?

_____

_____

E. Give the generalization for the total length of rods when you use rods of length l.

_____

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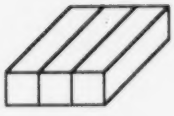
PATTERN 46

1

(No rods)

2



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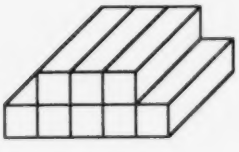

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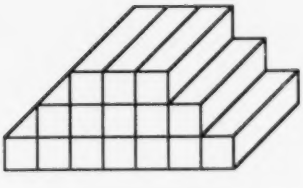

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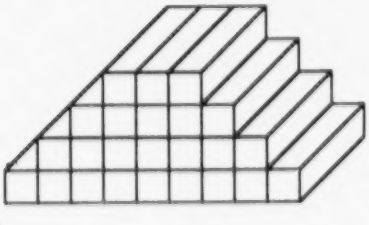

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n

Build the pattern three times, using light green, then purple, then yellow rods.

A. For the 38th design, how many rods are needed? _____

B. Which design will require exactly 1023 rods? _____

C. The generalization for the number of rods can be expressed as $(n^2 - 1)$ or $(n + 1)(n - 1)$. Describe how the rods can be rearranged to show clearly that both answers are correct.

_____

_____

_____

_____

D. How does the constant difference in any column relate to the generalization for the column?

_____

_____

_____

_____

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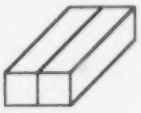
PATTERN 47

1



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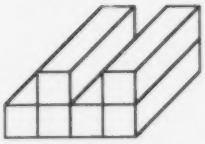

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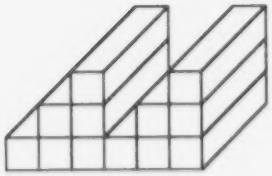

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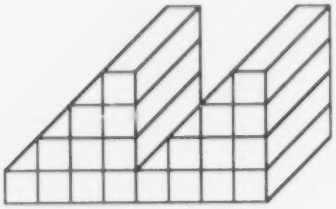

⋮

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n

Build the pattern three times, using dark green, then black, then brown rods.

A. How many rods will be needed for the 33rd design? _____

B. The _____ design will require exactly 1332 rods.

C. The generalization for the number of rods on this page is the same as for page 45. Describe one way you could rearrange the rods on this page so they look like the rod arrangements on page 45.

_____

_____

_____

_____

D. The generalization for the number of rods can be expressed as either $(n^2 + n)$ or $n(n + 1)$. Describe how you could rearrange the rods to show that $n(n + 1)$ is the correct answer.

_____

_____

_____

_____

E. How does the constant difference in each column relate to the generalization for that column?

_____

_____

_____

_____

55

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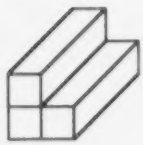
PATTERN 48

1

2



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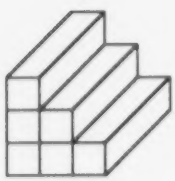

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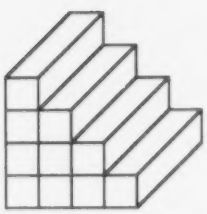

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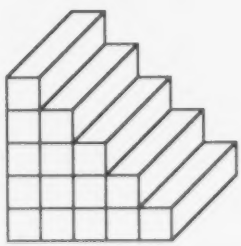

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n

Build the pattern three times, using light green, then purple, then yellow rods.

A. The 37th design will need how many rods? _____

B. The _____ design will need 946 rods.

C. How do the numbers grow in the columns on the Number of Rods page?

Number of Rods _____

Light Green _____

Purple _____

Yellow _____

When is there a constant difference?

_____

_____

_____

How much are the constant differences for the columns listed below?

Number of rods _____

Light green _____

Purple _____ Yellow _____

How does the growing pattern relate to the generalization for n?

_____

_____

_____

D. The generalization for the number of rods can be expressed as $1/2[n(n + 1)]$. This is $1/2$ of $n(n + 1)$. Show and describe how the rod arrangements are $1/2$ of $n(n + 1)$.

_____

_____

_____

56

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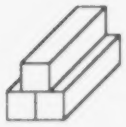
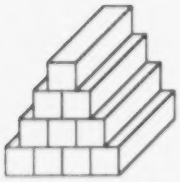
PATTERN 49

1



2



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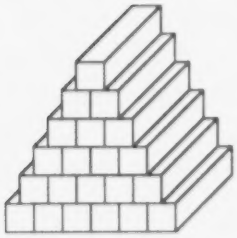

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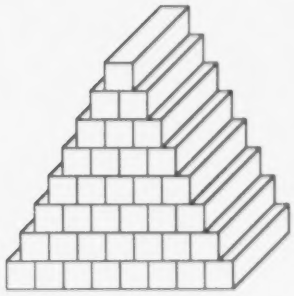

⋮

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n

Build the pattern three times, using purple, then yellow, then dark green rods.

A. The 34th design will need _____ rods.

B. Which design will need 1596 rods? _____

C. How do the numbers grow in the columns on the Number of Rods page?

Number of rods _____

Purple _____

Yellow _____

Dark green _____

When is there a constant difference?

_____

_____

_____

How much are the constant differences for the columns listed below?

Number of Rods _____ Purple _____

Yellow _____ Dark green _____

How does the constant difference relate to the generalization for n?

_____

_____

_____

D. Give the generalization for the total length of rods when you use rods of length l.

_____

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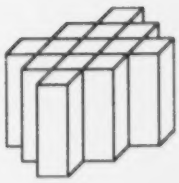
PATTERN 50

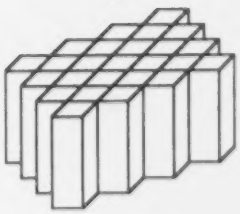
1

2

3



4



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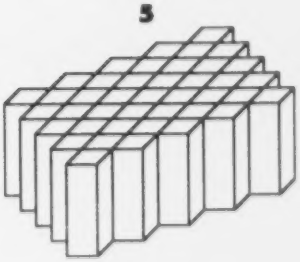

⋮

10

⋮

20

⋮

n

Build the pattern three times, using yellow, then dark green, then black rods.

A. How many rods will be needed for the 31st design? _____

B. The exact number of rods for the _____ design will be 2381.

C. How do the numbers grow in the columns of the Number of Rods page?

Number of Rods _____

Yellow _____

Dark Green _____

Black _____

When is there a constant difference?

_____

_____

_____

How much are the constant differences for the columns listed below?

Number of Rods _____ Yellow _____

Dark Green _____ Black _____

How do the constant differences relate to the generalization for n?

_____

_____

_____

D. Give the generalization for the total length of rods for rods of length l.

_____

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APPENDIX

Glossary of Terms

Building Algebraic Thinking uses a specific vocabulary in providing valuable prealgebra experiences to the student; it is important that the teacher become familiar with the terms used in the book.

coefficient of *n*: number by which the variable n is multiplied

constant difference (growing number): the difference in numbers of rods used between any two consecutive designs of a pattern

generalization: general equation describing the relationship between the ordinal number of a design (n) and the number of rods used for that design; the "rule" or formula for a pattern

linear function: generalization whose solutions for individual designs increase by a consistent number and whose equation can be plotted as a straight line

pattern: a series of an infinite number of consistent designs in which there is one possible arrangement and number of rods for each design; the pattern has a "rule" or generalized equation that can be used to determine the numbers of rods needed in any design

design: arrangement of rods that represents a stage or progression within a pattern (each activity page shows several designs of one overall pattern) each design requires a specific number and arrangement of rods consistent with the pattern

quadratic function: generalization involving the square of the variable; it will take on the general form of $ax^2 + bx + c$; graph of this function is a curved line (parabola)

variable: a symbol used as a place holder for a number that can vary in value

reverse processing: solving an equation in which the unknown (variable) is on the left side: instead of finding the number of rods used in a given pattern design ($2 \times 5 = ?$), the student is given the number of rods required and must determine the associated design ($2 \times ? = 10$) or the associated number of another kind of rod in the design

Teaching Guide

Building Algebraic Thinking is intended to be an exploratory/discovery based program; a formalistic teaching approach is not recommended. However, for those who will gain confidence with some initial structure, the guidelines below provide a questioning strategy that can be used as a model to get you started. Review them before teaching, and as you become familiar with the concepts, rely on your own judgment of what your students need moment-by-moment.

The order of activities for any pattern can be altered as needed: the student may copy and extend the pattern for several designs, fill in the associated numbers of rods, and answer one or more questions before completing every step in the guide. As you proceed, keep in mind the following points:

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- The steps below will familiarize you with the general concepts for the activity pages. In providing problems to the students, you may vary the types and extent of activities on each page; the important thing is that students progress, when they are ready, from the simple concrete experiences to the abstract concepts that naturally evolve.

- Students may need to just experiment and build many patterns before they are ready to formalize the concepts elicited through any questioning strategy. Be careful not to push them into a stage for which they are not yet ready. It is important for them to get comfortable with each new realization; in the process, don't worry if they seem to "regress" into a previous stage of simple rod building.

1. Building the designs

Have the students use the rods to copy the exact arrangement of objects for the designs shown on the activity page. As they proceed, ask questions to stimulate their analysis of the pattern:

What does this design resemble? How is it growing? What will happen as it grows?

2. Determining the numbers of rods

Ask students to notice the numbers of rods used for each pattern design and fill in the blank(s) of the Number of Rods page (pp. 55–57) for the section they are working in.

How many rods will we need for the first design? the second? the fifth?

3. Discovering the pattern

Allow the students to discover any number pattern occurring on the Number of Rods page they have filled out (including odd or even numbers) and the relationship between the design number and the number of rods used.

How are the numbers of rods used different from one design to the next? Do you notice any kind of pattern developing in the columns? How many rods do we add to each design? How do these "growing" numbers change? Are the differences always the same? Are they odd or even?

When the student can easily see how the numbers change regularly, you may want to use the term **constant difference**.

As the numbers grow, what is the difference occurring between designs? Is there a constant difference—is it the same difference from one design to the next throughout the pattern?

4. Extending the pattern

Encourage the students to "work forward" by having them predict the number of rods needed for subsequent designs, and then encourage reverse processing by having students predict what design number will require a given number of rods.

Can you predict how many rods will be needed in order to build the sixth and seventh designs?

How did you figure out the solution? How is the number of rods used related to the number of the design?

Which design will require a total of 86 rods? With 200 rods, which design could you build?

When the students are able to predict the answers for specific designs, encourage them to explain how they got their answers.

How did you know how many rods to use for design number 7? What was your formula? [e.g., 2 times the design number or 2×7]

Encourage the students to use mental arithmetic as they determine the rule for

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generalizing; let different students give the design number and the rest of the class give the end result.

Are there any numbers that will stump us? Can we find the answer for any whole number?

When students can easily give the number of rods, use *n* as a variable to represent any design number.

5. Generalizing with a variable

Encourage students to use a variable to create a generalization that can be used to determine the number of rods for any design in the pattern.

Is there an easy way to find the answer for any design of this particular pattern?

You may want to start using the term *generalize*.

You've already come up with some specific formulas for the different designs. How about a general formula that applies to the whole pattern? What if we let n *stand for any number? What do we do to* n? (The same as we did to the design number: $2 \times n$ = number of rods.)

Have students substitute any number for *n*.

Now, how can we find the total number of rods we need to build the 20th design? What do we do with the number 20? (Substitute 20 for *n*: $2 \times 20 = 40$ rods.)

When the generalization is found, it should be checked. Have students go back to the earlier, simpler designs and substitute design numbers for the variable to show that the generalization does indeed work.

6. Total length of rods (Section 3)

Have students build the first three or four designs—once with each of the three indicated colors of rods. As they fill in the Number of Rods page, the numbers in the Number of Rods column will be the same no matter which color rod is used. For each color rod listed in the other columns, students must compute the total length of rods.

If we use only dark green rods to build the pattern, each rod will be the same length (6 units—see p. viii for rod lengths). *How many of these rods are required for the first design?* (e.g., 2, as in Pattern 31) *What will be the total length of rods used for design 1?* ($2 \times 6 = 12$ units) *What about design 5? How many rods does it require?* (10) *What will be the total length of dark green rods for design 5?* ($10 \times 6 = 60$).

When the students are able to compute total lengths for given colors of rods, introduce the idea of a variable to substitute for rods of unspecified lengths.

We saw that to get the total length of rods, we multiply the number of rods needed times the length, so if we use purple rods (4 units each) *4 units times 10 rods equals a total length of 40 units. How can we describe the total length for 10 rods of an unknown type? What if we use "l" to stand for the length of any colored rods?* (The answer will be $10 \times l = 10l$.)

7. Analyzing and comparing physical arrangements (Section 3)

It is hoped that students will see that there are many different ways to solve problems. Students may come up with some very creative strategies, and these should be shared.

We have seen that constant difference may help students generalize. Another strategy they may find helpful is to analyze, compare, and rearrange the physical layout of rods.

Explain one way you can see the generalization for n *by looking at the physical arrangement of rods. How does the rod arrangement show that the generalization is true?* (Students may explore to discover the connection; for a generalization that

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includes $3n$, for example, they might look for 3 groups of n rods in the physical arrangement.)

What do you notice about the dimensions of the end view of the rod arrangement? Do you see any relationship to the generalization?

Students may find that the generalization is reflected in the end dimensions of the designs. In pattern 36, for example, the generalization $3n - 2$ is shown in the incomplete rectangle of $3 \times n$ squares (made up of the rod ends); however, one side of the rectangle is missing 2 squares. As the designs become more complex, it can become difficult to see the generalization in the physical arrangement, and the students may rely on the constant difference method. Whatever works for the student is fine.

Reproducible "Number of Rods" Pages

The Number of Rods pages (pp. 55–57) are to be reproduced and distributed so that students always have the correct sheet for the section they are working on. As the students build the rod designs, they should label the columns with the colors of rods required and record in the blanks the numbers of rods used for each design. In addition, for Section 3, they must record the total lengths of rods for each of three colors of rods. Use the chart below as a quick reference for specific information on the Number of Rods pages.

Number of Rods Page	Colors of Rods	Corresponding Patterns
Section 1 — p. 55	Two colors (shown in graphics on activity pages)	Patterns 1–15
Section 2 — p. 56	Three colors (shown in graphics on activity pages)	Patterns 16–30
Section 3 — p. 57	Three colors (listed on activity pages)	Patterns 31–50

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Section 1
NUMBER OF RODS

PATTERN _____

Design (Spans)	_____ Rods	_____ Rods	Rods (Total)
1			
2			
3			
4			
5			
• • •	• • •	• • •	• • •
10			
• • •	• • •	• • •	• • •
20			
• • •	• • •	• • •	• • •
n			

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Section 2
NUMBER OF RODS

PATTERN _____

Design (Spans)	_____ Rods	_____ Rods	_____ Rods	Rods (Total)
1				
2				
3				
4				
5				
• • •	• • •	• • •	• • •	• • •
10				
• • •	• • •	• • •	• • •	• • •
20				
• • •	• • •	• • •	• • •	• • •
n				

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Section 3
NUMBER OF RODS

Rod Lengths			
White	1	Dark Green	6
Red	2	Black	7
Light Green	3	Brown	8
Purple	4	Blue	9
Yellow	5	Orange	10

PATTERN _____

Design	Number of Rods	Total Length		
		_____ Rods	_____ Rods	_____ Rods
1				
2				
3				
4				
5				
⋮	⋮	⋮	⋮	⋮
10				
⋮	⋮	⋮	⋮	⋮
20				
⋮	⋮	⋮	⋮	⋮
n				

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ANSWER KEY

Note: The explanations included in the answer key are intended for the teacher (for clarification on how the answer *could* be found). It is not suggested that the students be given explanations or specific directions on how to find answers; students should be allowed to discover their own unique methods for solving the problems.

SECTION 1

Pattern 1 Answers

A. 30, 30
Spans are the dark green rods; $n = 30$.

B. 92
If $n = 46$, then $2n = 92$.

C. We add 2 to each number; they are the even numbers.

D. 49
If $2n = 98$, then $n = 49$.

E. No; 129 is an odd number; our total must always be an even number.

F. 558
If $n = 279$, then $2n = 558$.

Span	Dark Green	Light Green	Total
1	1	1	2
2	2	2	4
3	3	3	6
4	4	4	8
5	5	5	10
⋮	⋮	⋮	⋮
10	10	10	20
⋮	⋮	⋮	⋮
20	20	20	40
⋮	⋮	⋮	⋮
n	n	n	n + n or 2n

Pattern 2 Answers

A. 26, 52
If $n = 26$, then $2n = 52$.

B. 24
If $2n = 48$, then $n = 24$.

C. Yellow: We add 1 each time to get consecutive counting numbers.
Red: We add 2 each time and create the even numbers.
Total: We add 3 each time and create the multiples of 3.

D. 87 yellow: Yes; spans equal the yellow rods.
87 red: No; the number of red rods is always an even number.
120 total: Yes; 120 is divisible by 3. A 40-span bridge uses 120 rods.

E. 94
If $3n = 141$, then $n = 47$ and $2n = 94$.

F. 378, 567
Since the spans are the yellow rods, $n = 189$, $2n = 378$, and $3n = 567$.

Span	Yellow	Red	Total
1	1	2	3
2	2	4	6
3	3	6	9
4	4	8	12
5	5	10	15
⋮	⋮	⋮	⋮
10	10	20	30
⋮	⋮	⋮	⋮
20	20	40	60
⋮	⋮	⋮	⋮
n	n	n + n or 2n	n + n + n or 3n

Pattern 3 Answers

A. 33, 99
Since spans are the blue rods, $n = 33$ and $3n = 99$.

B. Blue: These are the consecutive counting numbers.
Red: These numbers go up by 3 and are the multiples of 3.
Total: These numbers go up by 4 and are the multiples of 4.

C. 78 blue: Yes; the number of spans equals the number of blue rods.
78 red: Yes; 78 is a multiple of 3. A 26-span bridge uses 78 red rods.

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78 total: No; 78 is not a multiple of 4.

D. Blue: The numbers grow by 1s.
Red: The numbers grow by 3s.
Total: The numbers grow by 4s.
The growing numbers are the numbers before n in the generalizations $1 \times n$, $3n$, and $4n$.

E. 189
If $4n = 252$, then $n = 63$ and $3n = 189$.

F. 477, 636
Since spans are blue rods, $n = 159$, $3n = 477$, and $4n = 636$.

Span	Blue	Red	Total
1	1	3	4
2	2	6	8
3	3	9	12
4	4	12	16
5	5	15	20
⋮	⋮	⋮	⋮
10	10	30	40
⋮	⋮	⋮	⋮
20	20	60	80
⋮	⋮	⋮	⋮
n	n	n + n + n or 3n	n + n + n + n or 4n

Pattern 4 Answers

A. 38, 39
Spans are black rods ($n = 38$); $n + 1 = 39$.

B. Black: The numbers grow by 1s.
Light green: The numbers grow by 1s.
Total: The numbers grow by 2s.
The growing numbers are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, and $2n + 1$.

C. No, the total will always be odd.
Since $2n$ is always even, $2n + 1$ must always be odd.

D. 59
If $2n + 1 = 119$, $n = 59$.

E. 375
Spans are black rods ($n = 187$); $2n + 1 = 375$.

F. When you add n and $n + 1$, you have two n's and a 1, so the answer is $2n + 1$.

Span	Black	Light Green	Total
1	1	2	3
2	2	3	5
3	3	4	7
4	4	5	9
5	5	6	11
⋮	⋮	⋮	⋮
10	10	11	21
⋮	⋮	⋮	⋮
20	20	21	41
⋮	⋮	⋮	⋮
n	n	n + 1	n + n + 1 or 2n + 1

Pattern 5 Answers

A. 35, 71
Spans are orange rods ($n = 35$); $2n + 1 = 71$.

B. Orange: The numbers grow by 1s.
Light green: The numbers grow by 2s.
Total: The numbers grow by 3s.
The growing numbers are the numbers before n in the generalizations $1 \times n$, $2n + 1$, and $3n + 1$.

C. 97 light green: Yes. We use only odd numbers of light green rods; the bridge with 48 spans will use 97.
97 total: Yes; the bridge with 32 spans will use 97 rods because $(3 \times 32) + 1 = 97$.

D. 95
If $3n + 1 = 142$, then $n = 47$ and $2n + 1 = 95$.

E. 175, 262
If $n = 87$, then $2n + 1 = 175$ and $3n + 1 = 262$.

F. When we add n and $2n + 1$ we have three n's and a 1, so the answer is $3n + 1$.

Span	Orange	Light Green	Total
1	1	3	4
2	2	5	7
3	3	7	10
4	4	9	13
5	5	11	16
⋮	⋮	⋮	⋮
10	10	21	31
⋮	⋮	⋮	⋮
20	20	41	61
⋮	⋮	⋮	⋮
n	n	2n + 1	3n + 1

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Pattern 6 Answers

Span	Blue	Red	Total
1	1	4	5
2	2	8	10
3	3	12	15
4	4	16	20
5	5	20	25
⋮	⋮	⋮	⋮
10	10	40	50
⋮	⋮	⋮	⋮
20	20	80	100
⋮	⋮	⋮	⋮
n	n	4n	5n

A. 39, 156
Spans are blue rods ($n = 39$); $4n = 156$.

B. Red: The numbers go up by 4 each time; these numbers are the multiples of 4.
Total: The numbers go up by 5; these are the multiples of 5.

C. 190
Since $4n = 152$, $n = 38$ and $5n = 190$.

D. 236 red: Yes (occurs with 59 spans); 236 is a multiple of 4.
236 total: No; 236 is not a multiple of 5.

E. Blue: The numbers grow by 1s.
Red: The numbers grow by 4s.
Total: The numbers grow by 5s.
The constant differences are the numbers before n in the answers $1 \times n$, $4n$, and $5n$.

F. 316, 395
Since $n = 79$, then $4n = 316$ and $5n = 395$.

G. When we add n and $4n$, we have a total of five n's, or $5n$.

Pattern 7 Answers

Span	Brown	Light Green	Total
1	1	4	5
2	2	7	9
3	3	10	13
4	4	13	17
5	5	16	21
⋮	⋮	⋮	⋮
10	10	31	41
⋮	⋮	⋮	⋮
20	20	61	81
⋮	⋮	⋮	⋮
n	n	3n + 1	4n + 1

A. 34, 103
Spans are brown rods ($n = 34$); $3n + 1 = 103$.

B. No. The total number of rods must always be odd because $4n + 1$ will always be odd.

C. 39
If $3n + 1 = 118$, then $n = 39$.

D. Brown: The numbers grow by 1s.
Light green: The numbers grow by 3s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $3n + 1$, and $4n + 1$.

E. 189 light green: No; 189 does not fit into our final answer of $3n + 1$ (there is no whole number n for which $3n + 1 = 189$).
189 total: Yes; $4n + 1 = 189$ is a valid equation ($n = 47$).

F. 262
Since $n = 87$, $3n + 1 = 262$.

G. Adding one n and three n's, we get four n's; we still have a 1, so the answer is $4n + 1$.

Pattern 8 Answers

A. 78, 78
Since $n = 39$, $2n = 78$ for both brown rods and white rods.

B. Brown: The numbers go up by 2s; these are the even numbers, or multiples of 2.
White: The numbers go up by 2s, and are the even numbers, or multiples of 2.
Total: The numbers go up by 4s, and are the multiples of 4.

C. 228
If $2n = 114$, $n = 57$ and $4n = 228$.

D. No. The number 155 is not a multiple of 4.

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E. Brown: The numbers grow by 2s.
White: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the generalizations $2n$, $2n$, and $4n$.

F. 438, 876
If $n = 219$, then $2n = 438$ and $4n = 876$.

G. When we add two n's and two n's, we have four n's, or $4n$.

Span	Brown	White	Total
1	2	2	4
2	4	4	8
3	6	6	12
4	8	8	16
5	10	10	20
⋮	⋮	⋮	⋮
10	20	20	40
⋮	⋮	⋮	⋮
20	40	40	80
⋮	⋮	⋮	⋮
n	2n	2n	4n

Pattern 9 Answers

A. 86, 129
Since n = 43, $2n = 86$ and $3n = 129$.

B. Blue: The numbers are the even numbers, or multiples of 2.
White: The numbers are the multiples of 3.
Total: The numbers are the multiples of 5.

C. 64
If $3n = 96$, then $n = 32$ and $2n = 64$.

D. 126 blue: Yes (occurs with 63 spans); 126 is an even number.
126 white: Yes (occurs with 42 spans); 126 is a multiple of 3.
126 total: No; 126 is not a multiple of 5.

E. Blue: The numbers grow by 2s.
White: The numbers grow by 3s.
Total: The numbers grow by 5s.
The constant differences are the numbers before n in the generalizations $2n$, $3n$, and $5n$.

F. 458, 687
If $n = 229$, then $2n = 458$ and $3n = 687$.

G. When we add two n's and three n's, we have five n's, or $5n$.

Span	Blue	White	Total
1	2	3	5
2	4	6	10
3	6	9	15
4	8	12	20
5	10	15	25
⋮	⋮	⋮	⋮
10	20	30	50
⋮	⋮	⋮	⋮
20	40	60	100
⋮	⋮	⋮	⋮
n	2n	3n	5n

Pattern 10 Answers

A. 48, 94
When $n = 47$, $n + 1 = 48$ and $2n = 94$.

B. 118 red: Yes; $n + 1 = 118$ when $n = 117$.
118 brown: Yes, 118 is an even number. We will use 118 brown rods when we have a bridge with 59 spans ($n = 59$).
118 total: Yes, $3n + 1 = 118$ when $n = 39$.

C. 58, 114
If $3n + 1 = 172$, then $n = 57$, $n + 1 = 58$, and $2n = 114$.

D. Red: The numbers grow by 1s.
Brown: The numbers grow by 2s.
Total: The numbers grow by 3s.
The constant differences are the numbers before n in the generalizations $(1 \times n) + 1$, $2n$, and $3n + 1$.

E. 474, 712
If $n = 237$, then $2n = 474$ and $3n + 1 = 712$.

F. Adding two n's and one n, we get three n's. We also have to add a 1 for a total of $3n + 1$.

Span	Red	Brown	Total
1	2	2	4
2	3	4	7
3	4	6	10
4	5	8	13
5	6	10	16
⋮	⋮	⋮	⋮
10	11	20	31
⋮	⋮	⋮	⋮
20	21	40	61
⋮	⋮	⋮	⋮
n	n + 1	2n	3n + 1

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Pattern 11 Answers

Span	Black	Red	Total
1	2	2	4
2	4	4	8
3	6	6	12
4	8	8	16
5	10	10	20
⋮	⋮	⋮	⋮
10	20	20	40
⋮	⋮	⋮	⋮
20	40	40	80
⋮	⋮	⋮	⋮
n	2n	2n	4n

A. 90, 90
If $n = 45$, then $2n = 90$ (the number of both black rods and red rods is $2n$).

B. Black: The numbers are the even numbers, or multiples of 2.
Red: The numbers are the even numbers, or multiples of 2.
Total: The numbers are the multiples of 4.

C. 114
If $4n = 228$, then $n = 57$ and $2n = 114$.

D. 214 red: Yes; 214 is an even number. A 107-span bridge will need 214 red rods.
214 total: No; 214 is not a multiple of 4.

E. Black: The numbers increase by 2s.
Red: The numbers increase by 2s.
Total: The numbers increase by 4s.
The constant differences are the numbers before n in the generalizations $2n$, $2n$, and $4n$.

F. 552
If $n = 276$, then $2n = 552$.

G. When we add two n's and two n's, we have four n's, or $4n$.

Pattern 12 Answers

Span	Orange	Red	Total
1	2	3	5
2	4	6	10
3	6	9	15
4	8	12	20
5	10	15	25
⋮	⋮	⋮	⋮
10	20	30	50
⋮	⋮	⋮	⋮
20	40	60	100
⋮	⋮	⋮	⋮
n	2n	3n	5n

A. 98, 147
If $n = 49$, then $2n = 98$ and $3n = 147$.

B. Orange: The numbers are even numbers, or multiples of 2.
Red: The numbers are multiples of 3.
Total: The numbers are multiples of 5.

C. 102, 153
If $5n = 255$, then $n = 51$, $2n = 102$, and $3n = 153$.

D. 135 orange: No; 135 is not an even number.
135 red: Yes; 135 is a multiple of 3. When $n = 45$ spans, $3n = 135$.
135 total: Yes; 135 is a multiple of 5. The bridge with 27 spans will need 135 rods.

E. Orange: The numbers grow by 2s.
Red: The numbers grow by 3s.
Total: The numbers grow by 5s.
The constant differences are the numbers before n in the generalizations $2n$, $3n$, and $5n$.

F. 582, 873
If $n = 291$, then $2n = 582$ and $3n = 873$.

G. When we add two n's and three n's, we have five n's, or $5n$.

Pattern 13 Answers

A. 48, 94
If $n = 47$, $n + 1 = 48$ and $2n = 94$.

B. 202 red: Yes; $n + 1 = 202$ red rods when $n = 201$ spans.
202 brown: Yes; 202 is even. The bridge with 101 spans will use 202 brown rods.
202 total: Yes; $3n + 1 = 202$ is valid. The bridge with 67 spans will use a total of 202 rods.

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C. 84, 166
If $3n + 1 = 250$, then $n = 83$, $n + 1 = 84$, and $2n = 166$.

D. Red: The numbers grow by 1s.
Brown: The numbers grow by 2s.
Total: The numbers grow by 3s.
The constant differences are the numbers before n in the generalizations $(1 \times n) + 1$, $2n$, and $3n + 1$.

E. 250, 498
If $n = 249$, then $n + 1 = 250$, and $2n = 498$.

F. Adding one n and two n's, we have three n's; we must also add a 1 for a total of $3n + 1$.

Span	Red	Brown	Total
1	2	2	4
2	3	4	7
3	4	6	10
4	5	8	13
5	6	10	16
⋮	⋮	⋮	⋮
10	11	20	31
⋮	⋮	⋮	⋮
20	21	40	61
⋮	⋮	⋮	⋮
n	n + 1	2n	3n + 1

Pattern 14 Answers

A. 78, 79
If $n = 39$, then $2n = 78$ and $2n + 1 = 79$.

B. 187 blue: No; 187 is not an even number.
187 red: Yes; 187 is an odd number. The bridge with 93 spans will use 187 red rods.
187 total: No; there is no whole number for n that fits $4n + 1 = 187$.

C. 132, 133
If $4n + 1 = 265$, $n = 66$, $2n = 132$, and $2n + 1 = 133$.

D. Blue: The numbers grow by 2s.
Red: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the answers $2n$, $2n + 1$, and $4n + 1$.

E. The sum of two even numbers is even and the sum of two odd numbers is even, but the sum of an even number and an odd number will always be odd. Since $2n$ is even and $2n + 1$ is odd, their sum will be an odd number.

F. 526, 527, 1053
If $n = 263$, then $2n = 526$, $2n + 1 = 527$, and $4n + 1 = 1053$.

Span	Blue	Red	Total
1	2	3	5
2	4	5	9
3	6	7	13
4	8	9	17
5	10	11	21
⋮	⋮	⋮	⋮
10	20	21	41
⋮	⋮	⋮	⋮
20	40	41	81
⋮	⋮	⋮	⋮
n	2n	2n + 1	4n + 1

Pattern 15 Answers

A. 111, 112
If $n = 37$, then $3n = 111$ and $3n + 1 = 112$.

B. 162 orange: Yes; 162 is a multiple of 3. The bridge with 54 spans needs 162 orange rods.
183 light green: No; no whole number for n solves $3n + 1 = 183$.
203 total: No; no whole number for n solves $6n + 1 = 203$.

C. 147, 148
If $6n + 1 = 295$, $n = 49$, $3n = 147$, and $3n + 1 = 148$.

D. Orange: The numbers grow by 3s.
Light green: The numbers grow by 3s.
Total: The numbers grow by 6s.
The constant differences are the numbers before n in the generalizations $3n$, $3n + 1$, and $6n + 1$.

E. In the Total generalization, $6n + 1$, $6n$ has to be even, so $6n + 1$ must be odd.

Span	Orange	Light Green	Total
1	3	4	7
2	6	7	13
3	9	10	19
4	12	13	25
5	15	16	31
⋮	⋮	⋮	⋮
10	30	31	61
⋮	⋮	⋮	⋮
20	60	61	121
⋮	⋮	⋮	⋮
n	3n	3n + 1	6n + 1

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F. 687, 688, 1375
If $n = 229$, then $3n = 687$, $3n + 1 = 688$, and $6n + 1 = 1375$.

SECTION 2

Pattern 16 Answers

Spans	Blue	White	Red	Total
1	1	1	2	4
2	2	2	4	8
3	3	3	6	12
4	4	4	8	16
5	5	5	10	20
⋮	⋮	⋮	⋮	⋮
10	10	10	20	40
⋮	⋮	⋮	⋮	⋮
20	20	20	40	80
⋮	⋮	⋮	⋮	⋮
n	n	n	2n	4n

A. 47, 47, 94
The number of spans = n; $n = 47$ for both blue and white rods; $2n = 94$.

B. Blue: The numbers go up by 1s; they are the counting numbers.
White: The numbers go up by 1s; they are the counting numbers.
Red: The numbers go up by 2s; they are the even numbers, or multiples of 2.
Total: The numbers go up by 4s; they are multiples of 4.

C. 59
If $4n = 236$, then $n = 59$.

D. No; 350 is not a multiple of 4.

E. Blue: The numbers grow by 1s.
White: The numbers grow by 1s.
Red: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the answers $1 \times n$, $1 \times n$, $2n$, and $4n$.

F. 792
If $n = 198$, then $4n = 792$.

G. When we add one n, one n, and two n's, we have a total of four n's, or $4n$.

Pattern 17 Answers

Spans	Brown	White	Light Green	Total
1	1	1	2	4
2	2	2	3	7
3	3	3	4	10
4	4	4	5	13
5	5	5	6	16
⋮	⋮	⋮	⋮	⋮
10	10	10	11	31
⋮	⋮	⋮	⋮	⋮
20	20	20	21	61
⋮	⋮	⋮	⋮	⋮
n	n	n	n + 1	3n + 1

A. 43, 43, 44
Spans, brown rods, and white rods all equal n; $n = 43$; therefore, $n + 1 = 44$.

B. 169
If $n + 1 = 57$, then $n = 56$ and $3n + 1 = 169$.

C. The totals increase by 3s.
The constant difference 3 is the number before n in the generalization $3n + 1$.

D. 57, 58
If $3n + 1 = 172$, then $n = 57$ and $n + 1 = 58$.

E. No; $3n + 1 = 204$ is not valid for any whole number (though 205 and 202 will work).

F. 622
If $n = 207$, then $3n + 1 = 622$.

G. Adding all the n's, we get three n's, or $3n$; we must then add the 1 for a sum of $3n + 1$.

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Pattern 18 Answers

Spans	Orange	Red	White	Total
1	1	2	2	5
2	2	4	4	10
3	3	6	6	15
4	4	8	8	20
5	5	10	10	25
⋮	⋮	⋮	⋮	⋮
10	10	20	20	50
⋮	⋮	⋮	⋮	⋮
20	20	40	40	100
⋮	⋮	⋮	⋮	⋮
n	n	2n	2n	5n

A. 45, 90, 90
We know that $n = 45$, so $2n = 90$.

B. Orange: The numbers are the counting numbers, or natural numbers.
Red: The numbers are even numbers, or multiples of 2.
White: The numbers are even numbers, or multiples of 2.
Total: The numbers are multiples of 5.

C. 195
If $2n = 78$, then $n = 39$ and $5n = 195$.

D. Orange: The numbers grow by 1s.
Red: The numbers grow by 2s.
White: The numbers grow by 2s.
Total: The numbers grow by 5s.
The constant differences occur before n in the generalizations $1 \times n$, $2n$, $2n$, and $5n$.

E. No; 204 is not a multiple of 5.

F. 59, 118, 118
If $5n = 295$, $n = 59$ and $2n = 118$.

G. 895
If $n = 179$, then $5n = 895$.

H. When we add all the n's, we have five n's, or $5n$.

Pattern 19 Answers

Spans	Blue	Red	White	Total
1	1	2	2	5
2	2	3	4	9
3	3	4	6	13
4	4	5	8	17
5	5	6	10	21
⋮	⋮	⋮	⋮	⋮
10	10	11	20	41
⋮	⋮	⋮	⋮	⋮
20	20	21	40	81
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	2n	4n + 1

A. 49, 50, 98
If $n = 49$, then $n + 1 = 50$ and $2n = 98$.

B. 193
If $2n = 96$, then $n = 48$ and $4n + 1 = 193$.

C. Blue: The numbers grow by 1s.
Red: The numbers grow by 1s.
White: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, $2n$, and $4n + 1$.

D. 292 total: No; the total must be odd ($4n + 1 = 292$ is invalid for any whole number).
293 total: Yes; $4n + 1 = 293$ is valid. We will use 293 rods when $n = 73$.

E. 90, 178
If $4n + 1 = 357$, $n = 89$, $n + 1 = 90$, and $2n = 178$.

F. 757
If $n = 189$, then $4n + 1 = 757$.

G. When we add all the n's, we have four n's or $4n$. Adding the 1, we get a sum of $4n + 1$.

Pattern 20 Answers

A. 48, 95, 190
If $n = 47$, $n + 1 = 48$, $2n + 1 = 95$, and $4n + 2 = 190$.

B. 113
If $n + 1 = 57$, $n = 56$ and $2n + 1 = 113$.

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C. Orange: The numbers grow by 1s.
Light green: The numbers grow by 1s.
Red: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, $2n + 1$, and $4n + 2$.

Spans	Orange	Light Green	Red	Total
1	1	2	3	6
2	2	3	5	10
3	3	4	7	14
4	4	5	9	18
5	5	6	11	22
⋮	⋮	⋮	⋮	⋮
10	10	11	21	42
⋮	⋮	⋮	⋮	⋮
20	20	21	41	82
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	2n + 1	4n + 2

D. 276 total: No; 276 does not equal $4n + 2$ for any whole number (though 274 and 278 work).
276 red: No; the number of red rods must be an odd number.

E. 87, 88, 175
If $4n + 2 = 350$, then $n = 87$, $n + 1 = 88$, and $2n + 1 = 175$.

F. Adding all the n's, we get four n's, or $4n$. We also add two 1s for a sum of $4n + 2$.

Pattern 21 Answers

A. 53, 54, 54, 161
The spans are the blue rods: $n = 53$, so $n + 1 = 54$ (for both light green and white rods) and $3n + 2 = 161$.

B. 146
If $n + 1 = 49$, then $n = 48$ and $3n + 2 = 146$.

C. Blue: The numbers grow by 1s.
Light green: The numbers grow by 1s.
White: The numbers grow by 1s.
Total: The numbers grow by 3s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, and $3n + 2$.

Spans	Blue	Light Green	White	Total
1	1	2	2	5
2	2	3	3	8
3	3	4	4	11
4	4	5	5	14
5	5	6	6	17
⋮	⋮	⋮	⋮	⋮
10	10	11	11	32
⋮	⋮	⋮	⋮	⋮
20	20	21	21	62
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	n + 1	3n + 2

D. Yes. A bridge with 65 spans ($n = 65$) will use a total of 197 rods ($3n + 2 = 197$).

E. 80
If $3n + 2 = 239$, then $n = 79$ and $n + 1 = 80$.

F. 166, 497
If $n = 165$, then $n + 1 = 166$ and $3n + 2 = 497$.

G. Adding all the n's, we get three n's, or $3n$; we add two 1s ($1 + 1 = 2$) for a sum of $3n + 2$.

Pattern 22 Answers

A. 46, 47, 94, 187
Spans are brown; $n = 46$, $n + 1 = 47$, $2n + 2 = 94$, and $4n + 3 = 187$.

B. 155
If $2n + 2 = 78$, then $n = 38$ and $4n + 3 = 155$.

C. Brown : The numbers grow by 1s.
Light green: The numbers grow by 1s.
White: The numbers grow by 2s.
Total: The numbers grow by 4s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, $2n + 2$, and $4n + 3$.

Spans	Brown	Light Green	White	Total
1	1	2	4	7
2	2	3	6	11
3	3	4	8	15
4	4	5	10	19
5	5	6	12	23
⋮	⋮	⋮	⋮	⋮
10	10	11	22	43
⋮	⋮	⋮	⋮	⋮
20	20	21	42	83
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	2n + 2	4n + 3

D. No; 253 does not equal $4n + 3$ for any whole number.

E. 160

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If $4n + 3 = 319$, then $n = 79$ and $2n + 2 = 160$.

F. 368, 735
Since $n = 183$, $2n + 2 = 368$ and $4n + 3 = 735$.

G. Adding the n's, we get four n's, or $4n$; adding the numbers, we get 3, for a sum of $4n + 3$.

Pattern 23 Answers

Spans	Black	Red	White	Total
1	1	4	2	7
2	2	8	4	14
3	3	12	6	21
4	4	16	8	28
5	5	20	10	35
⋮	⋮	⋮	⋮	⋮
10	10	40	20	70
⋮	⋮	⋮	⋮	⋮
20	20	80	40	140
⋮	⋮	⋮	⋮	⋮
n	n	4n	2n	7n

A. 48, 192, 96, 336
Spans are black: $n = 48$, $4n = 192$, $2n = 96$, and $7n = 336$.

B. Black: The numbers are the whole numbers.
Red: The numbers are the multiples of 4.
White: The numbers are even numbers, or multiples of 2.
Total: The numbers are the multiples of 7.

C. 273
Since $4n = 156$, $n = 39$ and $7n = 273$.

D. 186 white: Yes; $2n = 186$ white rods when there are 93 spans.
186 red: No; 186 does not equal $4n$ for any whole number.

E. Black: The numbers grow by 1s.
Red: The numbers grow by 4s.
White: The numbers grow by 2s.
Total: The numbers grow by 7s.
These constant differences are the numbers before n in the answers $1 \times n$, $4n$, $2n$, and $7n$.

F. 212, 106
If $7n = 371$, then $n = 53$, $4n = 212$, and $2n = 106$.

G. 197, 788, 394, 1379
Since $n = 197$ (spans = n), $4n = 788$, $2n = 394$, and $7n = 1379$.

Pattern 24 Answers

Spans	Blue	Light Green	White	Total
1	1	4	2	7
2	2	6	3	11
3	3	8	4	15
4	4	10	5	19
5	5	12	6	23
⋮	⋮	⋮	⋮	⋮
10	10	22	11	43
⋮	⋮	⋮	⋮	⋮
20	20	42	21	83
⋮	⋮	⋮	⋮	⋮
n	n	2n + 2 or 2(n + 1)	n + 1	4n + 3

A. 51, 104, 52, 207
Since $n = 51$ (spans = n), $2n + 2 = 104$, $n + 1 = 52$, and $4n + 3 = 207$.

B. 135
If $2n + 2 = 68$, then $n = 33$ and $4n + 3 = 135$.

C. Blue: The numbers grow by 1s.
White: The numbers grow by 1s.
Light green: The numbers grow by 2s.
Total: The numbers grow by 4s.
These constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, $2n + 2$, and $4n + 3$.

D. 235 light green: No; the number of light green rods is always even; 235 does not equal $2n + 2$ for any whole number.
235 total: Yes; $4n + 3 = 235$ rods when there are 58 spans.

E. 68
If $4n + 3 = 271$, then $n = 67$ and $n + 1 = 68$.

F. 392, 783
If $n = 195$, then $2n + 2 = 392$ and $4n + 3 = 783$.

G. Combining all the n's, we get four n's or $4n$. When we add the 2 and the 1, we get $4n + 3$.

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Pattern 25 Answers

Spans	Orange	Light Green	White	Total
1	1	5	2	8
2	2	8	3	13
3	3	11	4	18
4	4	14	5	23
5	5	17	6	28
⋮	⋮	⋮	⋮	⋮
10	10	32	11	53
⋮	⋮	⋮	⋮	⋮
20	20	62	21	103
⋮	⋮	⋮	⋮	⋮
n	n	3n + 2	n + 1	5n + 3

A. 49, 149, 50, 248
Since $n = 49$, $3n + 2 = 149$, $n + 1 = 50$, and $5n + 3 = 248$.

B. 188
If $3n + 2 = 113$, then $n = 37$ and $5n + 3 = 188$.

C. Orange: The numbers grow by 1s.
White: The numbers grow by 1s.
Light green: The numbers grow by 3s.
Total: The numbers grow by 5s.
The constant differences are the numbers before n in the generalizations $1 \times n$, $(1 \times n) + 1$, $3n + 2$, and $5n + 3$.

D. 213 light green: No; 213 does not equal $3n + 2$ for any whole number.
213 total: Yes; $5n + 3 = 213$ rods when the bridge has 42 spans.

E. 58, 173
If $5n + 3 = 288$, then $n = 57$, $n + 1 = 58$, and $3n + 2 = 173$.

F. 838
If $n = 167$, then $5n + 3 = 838$.

G. Combining the n's, we get five n's, or $5n$. Adding the 2 and 1 gives $5n + 3$.

Pattern 26 Answers

Spans	Black	Red	White	Total
1	2	2	2	6
2	4	4	4	12
3	6	6	6	18
4	8	8	8	24
5	10	10	10	30
⋮	⋮	⋮	⋮	⋮
10	20	20	20	60
⋮	⋮	⋮	⋮	⋮
20	40	40	40	120
⋮	⋮	⋮	⋮	⋮
n	2n	2n	2n	6n

A. 114, 114, 114, 342
If $n = 57$, then $2n = 114$ and $6n = 342$.

B. Black: The numbers are even numbers, or multiples of 2.
Red: The numbers are even numbers, or multiples of 2.
White: The numbers are even numbers, or multiples of 2.
Total: The numbers are the multiples of 6.

C. 258
If $2n = 86$, $n = 43$ and $6n = 258$.

D. Black: The numbers grow by 2s.
Red: The numbers grow by 2s.
White: The numbers grow by 2s.
Total: The numbers grow by 6s.
The constant differences are the numbers in front of the n in the generalizations $2n$, $2n$, $2n$, and $6n$.

E. 292 white: Yes; $2n = 292$ white rods when the bridge has 146 spans.
292 total: No; 292 is not a multiple of 6.

F. 118
If $6n = 354$, then $n = 59$ and $2n = 118$.

G. 518, 1554
If $n = 259$, then $2n = 518$ and $6n = 1554$.

H. When we combine all the n's, we have six n's, or $6n$.

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Pattern 27 Answers

Spans	Brown	Light Green	Red	Total
1	2	2	2	6
2	4	3	4	11
3	6	4	6	16
4	8	5	8	21
5	10	6	10	26
⋮	⋮	⋮	⋮	⋮
10	20	11	20	51
⋮	⋮	⋮	⋮	⋮
20	40	21	40	101
⋮	⋮	⋮	⋮	⋮
n	2n	n + 1	2n	5n + 1

A. 92, 47, 92, 231
If $n = 46$, then $2n = 92$, $n + 1 = 47$, and $5n + 1 = 231$.

B. 186
If $2n = 74$, then $n = 37$ and $5n + 1 = 186$.

C. Brown: The numbers grow by 2s.
Red: The numbers grow by 2s.
Light green: The numbers grow by 1s.
Total: The numbers grow by 5s.
The constant differences are the numbers in front of n in the generalizations $2n$, $2n$, $(1 \times n) + 1$, and $5n + 1$.

D. 121 light green: Yes; the bridge with 120 spans will use 121 light green rods.
121 red: No; 121 is not an even number.
121 total: Yes; $5n + 1 = 121$ rods when the bridge has 24 spans.

E. 174
If $5n + 1 = 436$, then $n = 87$ and $2n = 174$.

F. 438, 220, 1096
If $n = 219$, then $2n = 438$, $n + 1 = 220$, and $5n + 1 = 1096$.

G. Combining all the n's, we get five n's, or $5n$. We add the number 1 for a sum of $5n + 1$.

Pattern 28 Answers

Spans	Blue	Red	White	Total
1	2	5	1	8
2	4	8	2	14
3	6	11	3	20
4	8	14	4	26
5	10	17	5	32
⋮	⋮	⋮	⋮	⋮
10	20	32	10	62
⋮	⋮	⋮	⋮	⋮
20	40	62	20	122
⋮	⋮	⋮	⋮	⋮
n	2n	3n + 2	n	6n + 2

A. 102, 155, 51, 308
If $n = 51$, then $2n = 102$, $3n + 2 = 155$, and $6n + 2 = 308$.

B. 176
If $3n + 2 = 89$, then $n = 29$ and $6n + 2 = 176$.

C. Blue: The numbers grow by 2s.
Red: The numbers grow by 3s.
White: The numbers grow by 1s.
Total: The numbers grow by 6s.
These constant differences are the numbers before n in the generalizations $2n$, $3n + 2$, $1 \times n$, and $6n + 2$.

D. 119 red: Yes; $3n + 2 = 119$ red rods when the bridge has 39 spans.
119 total: No; 119 does not equal $6n + 2$ for any whole number.

E. 98
If $6n + 2 = 296$, then $n = 49$ and $2n = 98$.

F. 374, 563, 187, 1124
If $n = 187$, then $2n = 372$, $3n + 2 = 563$, and $6n + 2 = 1124$.

G. When we combine the n's, we have six n's, or $6n$. We also have the 2 for a sum of $6n + 2$.

Pattern 29 Answers

A. 108, 162, 164, 434
If $n = 54$, then $2n = 108$, $3n = 162$, $3n + 2 = 164$, and $8n + 2 = 434$.

B. 226
If $3n = 84$, then $n = 28$ and $8n + 2 = 226$.

C. White: The numbers grow by 2s.

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Orange: The numbers grow by 3s.
Light green: The numbers grow by 3s.
Total: The numbers grow by 8s.
The constant differences come before n in the generalizations $2n$, $3n$, $3n + 2$, and $8n + 2$.

Spans	White	Orange	Light Green	Total
1	2	3	5	10
2	4	6	8	18
3	6	9	11	26
4	8	12	14	34
5	10	15	17	42
⋮	⋮	⋮	⋮	⋮
10	20	30	32	82
⋮	⋮	⋮	⋮	⋮
20	40	60	62	162
⋮	⋮	⋮	⋮	⋮
n	2n	3n	3n + 2	8n + 2

D. 130 orange: No; 130 is not a multiple of 3 and does not equal $3n$ for any whole number.
538 total: Yes; $8n + 2 = 538$ rods when the bridge has 67 spans.

E. 149
If $8n + 2 = 394$, then $n = 49$ and $3n + 2 = 149$.

F. 278, 417, 419, 1114
If $n = 139$, then $2n = 278$, $3n = 417$, $3n + 2 = 419$, and $8n + 2 = 1114$.

Pattern 30 Answers

A. 60, 177, 237, 474
If $n = 59$, then $n + 1 = 60$, $3n = 177$, $4n + 1 = 237$, and $8n + 2 = 474$.

B. 258
If $3n = 96$, $n = 32$ and $8n + 2 = 258$.

C. Light green: The numbers grow by 1s.
Blue: The numbers grow by 3s.
Red: The numbers grow by 4s.
Total: The numbers grow by 8s.
The constant differences are the numbers in front of n in the generalizations $(1 \times n) + 1$, $3n$, $4n + 1$, and $8n + 2$.

Spans	Light Green	Blue	Red	Total
1	2	3	5	10
2	3	6	9	18
3	4	9	13	26
4	5	12	17	34
5	6	15	21	42
⋮	⋮	⋮	⋮	⋮
10	11	30	41	82
⋮	⋮	⋮	⋮	⋮
20	21	60	81	162
⋮	⋮	⋮	⋮	⋮
n	n + 1	3n	4n + 1	8n + 2

D. 269 red: Yes; $4n + 1 = 269$ red rods when the bridge has 67 spans.
572 total: No; 572 does not equal $8n + 2$ for any whole number.

E. 317
If $8n + 2 = 634$, then $n = 79$ and $4n + 1 = 317$.

F. Combining the n's, we get eight n's, or $8n$. We add the two 1s to get $8n + 2$.

SECTION 3

Pattern 31 Answers

A. 118
If $n = 59$, then $2n = 118$.

B. Number of rods: The numbers grow by 2s.
Yellow: The numbers grow by 10s.
Purple: The numbers grow by 8s.
Dark green: The numbers grow by 12s.
The constant differences are the numbers in front of n in the generalizations $2n$, $10n$, $8n$, and $12n$.

C. Black: $7(2n) = 14n$ (blacks rods are 7 units long)
Brown: $8(2n) = 16n$ (brown rods are 8 units long)
Rods of length l: $l(2n) = 2ln$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Dark Green
1	2	10	8	12
2	4	20	16	24
3	6	30	24	36
4	8	40	32	48
5	10	50	40	60
⋮	⋮	⋮	⋮	⋮
10	20	100	80	120
⋮	⋮	⋮	⋮	⋮
20	40	200	160	240
⋮	⋮	⋮	⋮	⋮
n	2n	5(2n) or 10n	4(2n) or 8n	6(2n) or 12n

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Pattern 32 Answers

A. 189
If $n = 63$, $3n = 189$.

B. Number of rods: The numbers grow by 3s.
Yellow: The numbers grow by 15s.
Purple: The numbers grow by 12s.
Dark green: The numbers grow by 18s.
The constant differences are the numbers in front of n in the generalizations $3n$, $15n$, $12n$, and $18n$.

C. Black: $7(3n) = 21n$ (black rods are 7 units long)
Brown: $8(3n) = 24n$ (brown rods are 8 units long)
Rods of length l: $l(3n) = 3ln$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Dark Green
1	3	15	12	18
2	6	30	24	36
3	9	45	36	54
4	12	60	48	72
5	15	75	60	90
⋮	⋮	⋮	⋮	⋮
10	30	150	120	180
⋮	⋮	⋮	⋮	⋮
20	60	300	240	360
⋮	⋮	⋮	⋮	⋮
n	3n	5(3n) or 15n	4(3n) or 12n	6(3n) or 18n

Pattern 33 Answers

A. 330
If $n = 66$, $5n = 330$.

B. No; 142 does not equal $5n$ for any whole number n; 142 is not a multiple of 5.

C. Number of rods: The numbers grow by 5s.
Yellow: The numbers grow by 25s.
Purple: The numbers grow by 20s.
Dark green: The numbers grow by 30s.
The constant differences are the numbers in front of n in the generalizations $5n$, $25n$, $20n$, and $30n$.

D. Black: $7(5n) = 35n$ (black rods are 7 units long)
Brown: $8(5n) = 40n$ (brown rods are 8 units long)
Rods of length l: $l(5n) = 5ln$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Dark Green
1	5	25	20	30
2	10	50	40	60
3	15	75	60	90
4	20	100	80	120
5	25	125	100	150
⋮	⋮	⋮	⋮	⋮
10	50	250	200	300
⋮	⋮	⋮	⋮	⋮
20	100	500	400	600
⋮	⋮	⋮	⋮	⋮
n	5n	5(5n) or 25n	4(5n) or 20n	6(5n) or 30n

Pattern 34 Answers

A. 292
If $n = 73$, then $4n = 292$.

B. No; 346 does not equal $4n$ for any whole number n; 346 is not a multiple of 4.

C. Number of rods: The numbers grow by 4s.
Yellow: The numbers grow by 20s.
Purple: The numbers grow by 16s.
Dark green: The numbers grow by 24s.
The constant differences are the numbers in front of n in the generalizations $4n$, $20n$, $16n$, and $24n$.

D. Black: $7(4n) = 28n$ (black rods are 7 units long)
Brown: $8(4n) = 32n$ (brown rods are 8 units long)
Rods of length l: $l(4n) = 4ln$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Dark Green
1	4	20	16	24
2	8	40	32	48
3	12	60	48	72
4	16	80	64	96
5	20	100	80	120
⋮	⋮	⋮	⋮	⋮
10	40	200	160	240
⋮	⋮	⋮	⋮	⋮
20	80	400	320	480
⋮	⋮	⋮	⋮	⋮
n	4n	5(4n) or 20n	4(4n) or 16n	6(4n) or 24n

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Pattern 35 Answers

A. 141
If n 71, then $2n - 1 = 141$.

B. No. The number of rods is always an odd number.

C. When we multiply 4 in $4(2n - 1)$ times $(2n - 1)$, we get $8n - 4$; i.e., $4(2n - 1) = (4 \times 2n) + (4 \times -1) = 8n - 4$.
Or, when we divide both $8n$ and -4 by 4, we get $2n$ and -1; i.e., $8n - 4 = 4(2n - 1)$.

D. Black: $7(2n - 1) = 14n - 7$ (black rods are 7 units long)
Brown: $8(2n - 1) = 16n - 8$ (brown rods are 8 units long)
Rods of length l: $l(2n - 1) = 2ln - l$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Dark Green
1	1	5	4	6
2	3	15	12	18
3	5	25	20	30
4	7	35	28	42
5	9	45	36	54
⋮	⋮	⋮	⋮	⋮
10	19	95	76	114
⋮	⋮	⋮	⋮	⋮
20	39	195	156	234
⋮	⋮	⋮	⋮	⋮
n	2n – 1	5(2n – 1) or 10n – 5	4(2n – 1) or 8n – 4	6(2n – 1) or 12n – 6

Pattern 36 Answers

A. 202
If $n = 68$, then $3n - 2 = 202$.

B. When we multiply 3 in $3(3n - 2)$ by $(3n - 2)$, we get $9n - 6$; i.e., $3(3n - 2) = (3 \times 3n) + (3 \times -2) = 9n - 6$.
Or, when we divide both $9n$ and -6 by 3, we get $3n$ and -2; i.e., $9n - 6 = 3(3n - 2)$.

C. If two rods are added to each design, the rectangle shape of each end will have the dimensions $3 \times n$, or $3n$.

D. Dark green: $6(3n - 2) = 18n - 12$ (dark green rods are 6 units long)
Black: $7(3n - 2) = 21n - 14$ (black rods are 7 units long)
Rods of length l: $l(3n - 2) = 3ln - 2l$

Design Number	Number of Rods	Total Length		
		Yellow	Light Green	Purple
1	1	5	3	4
2	4	20	12	16
3	7	35	21	28
4	10	50	30	40
5	13	65	39	52
⋮	⋮	⋮	⋮	⋮
10	28	140	84	112
⋮	⋮	⋮	⋮	⋮
20	58	290	174	232
⋮	⋮	⋮	⋮	⋮
n	3n – 2	5(3n – 2) or 15n – 10	3(3n – 2) or 9n – 6	4(3n – 2) or 12n – 8

Pattern 37 Answers

A. 151
If $n = 74$, then $2n + 3 = 151$.

B. Yes; $2n + 3 = 121$ rods when $n = 59$.

C. When we multiply 7 by $(2n + 3)$, we get $7 \times 2n = 14n$ and $7 \times 3 = 21$ for a total of $14n + 21$.
Or, when we divide both $14n$ and 21 by 7, we get $\frac{14n + 21}{7} = 2n + 3$.

D. 87th
If $2n + 3 = 177$, then $n = 87$.

E. Light green: $3(2n + 3) = 6n + 9$ (light green rods are 3 units long)
Dark green: $6(2n + 3) = 12n + 18$ (dark green rods are 6 units long)
Rods of length l: $l(2n + 3) = 2ln + 3l$

Design Number	Number of Rods	Total Length		
		Yellow	Purple	Black
1	5	25	20	35
2	7	35	28	49
3	9	45	36	63
4	11	55	44	77
5	13	65	52	91
⋮	⋮	⋮	⋮	⋮
10	23	115	92	161
⋮	⋮	⋮	⋮	⋮
20	43	215	172	301
⋮	⋮	⋮	⋮	⋮
n	2n + 3	5(2n + 3) or 10n + 15	4(2n + 3) or 8n + 12	7(2n + 3) or 14n + 21

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Pattern 38 Answers

A. 137
If $n = 69$, then $2n - 1 = 137$.

B. The rods on this page can be rearranged into the pattern shown on page 35. Any rods above the bottom level in the first column can be placed to the right on the rods of the bottom level to make two layers.

C. 100th
If $2n - 1 = 199$, then $n = 100$.

D. When we multiply 5 by $(2n - 1)$, we get $5 \times 2n = 10n$ and $5 \times (-1) = -5$, for a total of $10n - 5$.
Or, dividing both $10n$ and -5 by 5, we get $\frac{10n-5}{5} = 2n - 1$.

E. For rods of length l, the generalization is $l(2n - 1) = 2ln - l$.

Design Number	Number of Rods	Total Length		
		Yellow	Black	Blue
1	1	5	7	9
2	3	15	21	27
3	5	25	35	45
4	7	35	49	63
5	9	45	63	81
⋮	⋮	⋮	⋮	⋮
10	19	95	133	171
⋮	⋮	⋮	⋮	⋮
20	39	195	273	351
⋮	⋮	⋮	⋮	⋮
n	2n - 1	5(2n - 1) or 10n - 5	7(2n - 1) or 14n - 7	9(2n - 1) or 18n - 9

Pattern 39 Answers

A. 220
If $n = 74$, $3n - 2 = 220$.

B. On page 39, the single rods on the bottom level can always be placed on top of the rods in the next column to the right; by pushing the columns together, you create a rectangular solid (with height of three) with one rod left over on the right side.

C. 83
If $3n - 2 = 247$, then $n = 83$.

D. Black: $7(3n - 2)$ is the same as $(7 \times 3n) + [7 \times (-2)] = 21n + (-14) = 21n - 14$.
Brown: $8(3n - 2)$ is the same as $(8 \times 3n) + [8 \times (-2)] = 24n + (-16) = 24n - 16$.
Blue: $9(3n - 2)$ is the same as $(9 \times 3n) + [9 \times (-2)] = 27n - 18$.

E. For rods of length l, the generalization is $l(3n - 2)$, or $3ln - 2l$.

Design Number	Number of Rods	Total Length		
		Black	Brown	Blue
1	1	7	8	9
2	4	28	32	36
3	7	49	56	63
4	10	70	80	90
5	13	91	104	117
⋮	⋮	⋮	⋮	⋮
10	28	196	224	252
⋮	⋮	⋮	⋮	⋮
20	58	406	464	522
⋮	⋮	⋮	⋮	⋮
n	3n - 2	7(3n - 2) or 21n - 14	8(3n - 2) or 24n - 16	9(3n - 2) or 27n - 18

Pattern 40 Answers

A. 126
If $n = 62$, $2n + 2 = 126$.

B. You could move the top rod in the first column (page 37) to the top of the middle bottom rod. Then you could move the top rod in the last column to the bottom layer to make the arrangement resemble the pattern on page 40.

C. 77
If $2n + 2 = 156$, then $n = 77$.

D. Purple: $4(2n + 2)$ is $(4 \times 2n) + (4 \times 2) = 8n + 8$.
Dark green: $6(2n + 2)$ is $(6 \times 2n) + (6 \times 2) = 12n + 12$.
Black: $7(2n + 2)$ is $(7 \times 2n) + (7 \times 2) = 14n + 14$.

E. The generalization is $l(2n + 2)$, or $2ln + 2l$.

Design Number	lumber of Rods	Total Length		
		Purple	Dark Green	Black
1	4	16	24	28
2	6	24	36	42
3	8	32	48	56
4	10	40	60	70
5	12	48	72	84
⋮	⋮	⋮	⋮	⋮
10	22	88	132	154
⋮	⋮	⋮	⋮	⋮
20	42	168	252	294
⋮	⋮	⋮	⋮	⋮
n	2n + 2 or 2(n + 1)	4(2n + 2) or 8n + 8	6(2n + 2) or 12n + 12	7(2n + 2) or 14n + 14

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Pattern 41 Answers

A. 239
If $n = 79$, then $3n + 2 = 239$.

B. 91
If $3n + 2 = 275$, then $n = 91$.

C. You can add a new column of two rods at the left to make the block pattern on page 32 the same as the one on page 41. You can think of the two extra rods as the 2 in $3n + 2$.

D. Dark green: $6(3n + 2)$ is $(6 \times 3n) + (6 \times 2)$, or $18n + 12$.
Brown: $8(3n + 2)$ is $(8 \times 3n) + (8 \times 2)$, or $24n + 16$.
Blue: $9(3n + 2)$ is $(9 \times 3n) + (9 \times 2)$, or $27n + 18$.

E. The generalization is $l(3n + 2)$, or $3ln + 2l$.

Design Number	Number of Rods	Total Length		
		Dark Green	Brown	Blue
1	5	30	40	45
2	8	48	64	72
3	11	66	86	99
4	14	84	112	126
5	17	102	136	153
⋮	⋮	⋮	⋮	⋮
10	32	192	256	288
⋮	⋮	⋮	⋮	⋮
20	62	372	496	558
⋮	⋮	⋮	⋮	⋮
n	3n + 2	6(3n + 2) or 18n + 12	8(3n + 2) or 24n + 16	9(3n + 2) or 27n + 18

Pattern 42 Answers

A. 331
If $n = 66$, then $5n + 1 = 331$.

B. 73rd
If $5n + 1 = 366$, then $n = 73$.

C. You could place the single rods of the bottom level (except the last one on the right) on top of the rods in the column to their right; push the columns together.

D. Light green: $3(5n + 1)$ is $(3 \times 5n) + (3 \times 1)$, or $15n + 3$.
Yellow: $5(5n + 1)$ is $(5 \times 5n) + (5 \times 1)$, or $25n + 5$.
Brown: $8(5n + 1)$ is $(8 \times 5n) + (8 \times 1)$, or $40n + 8$.

E. The generalization is $l(5n + 1)$, or $5ln + l$.

Design Number	Number of Rods	Total Length		
		Light Green	Yellow	Brown
1	6	18	30	48
2	11	33	55	88
3	16	48	80	128
4	21	63	105	168
5	26	78	130	208
⋮	⋮	⋮	⋮	⋮
10	51	153	255	408
⋮	⋮	⋮	⋮	⋮
20	101	303	505	808
⋮	⋮	⋮	⋮	⋮
n	5n + 1	3(5n + 1) or 15n + 3	5(5n + 1) or 25n + 5	8(5n + 1) or 40n + 8

Pattern 43 Answers

A. 1156
If $n = 34$, then $n^2 = 1156$.

B. 41
If $n^2 = 1681$, then $n = 41$.

C. The first time we subtract numbers in a column, the amounts vary (see Differences table, below right).
Number of rods: 3, 5, 7, 9...
Light green: 9, 15, 21, 27...
Purple: 12, 20, 28, 36...
Yellow: 15, 25, 35, 45...
We find a constant difference when we subtract the differences; for example, the original differences in the yellow column change by 10 each time. (The Differences table shows the differences between column numbers as well as the constant differences between the original differences.) The constant differences for the four columns are 2 (number of rods), 6 (light green), 8 (purple), and 10 (yellow). The constant differences divided by 2 are the numbers in front of n^2 in the generalizations $1 \times n^2$, $3n^2$, $4n^2$, $5n^2$.

Design Number	Number of Rods	Total Length		
		Light Green	Purple	Yellow
1	1	3	4	5
2	4	12	16	20
3	9	27	36	45
4	16	48	64	80
5	25	75	100	125
⋮	⋮	⋮	⋮	⋮
10	100	300	400	500
⋮	⋮	⋮	⋮	⋮
20	400	1200	1600	2000
⋮	⋮	⋮	⋮	⋮
n	n^2	$3n^2$	$4n^2$	$5n^2$

Differences			
Number of Rods	Dark Green	Purple	Yellow
3	9	12	15
> 2	> 6	> 8	> 10
5	15	20	25
> 2	> 6	> 8	> 10
7	21	28	35
> 2	> 6	> 8	> 10
9	27	36	45

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D. Dark green: $6n^2$
Black: $7n^2$
Rods of length l: ln^2

Pattern 44 Answers

A. 1370
If $n = 37$, then $n^2 + 1 = 1370$.

B. 29th
If $n^2 + 1 = 842$, then $n = 29$.

C. The first time we subtract numbers in a column, the amounts vary (see Differences table, below right). We find a constant difference when we subtract the differences themselves. (The Differences table shows the differences between column numbers as well as the constant differences between the original differences.) The constant differences are 2, 10, 12, and 14 for the four columns. The constant differences divided by 2 are the numbers in front of n^2 in the generalizations.

D. Brown: $8(n^2 + 1) = 8n^2 + 8$
Blue: $9(n^2 + 1) = 9n^2 + 9$
Rods of length l: $l(n^2 + 1) = ln^2 + l$

Design Number	Number of Rods	Total Length		
		Yellow	Dark Green	Black
1	2	10	12	14
2	5	25	30	35
3	10	50	60	70
4	17	85	102	119
5	26	130	156	182
⋮	⋮	⋮	⋮	⋮
10	101	505	606	707
⋮	⋮	⋮	⋮	⋮
20	401	2005	2406	2807
⋮	⋮	⋮	⋮	⋮
n	$n^2 + 1$	$5(n^2 + 1)$ or $5n^2 + 5$	$6(n^2 + 1)$ or $6n^2 + 6$	$7(n^2 + 1)$ or $7n^2 + 7$

Differences			
Number of Rods	Yellow	Dark Green	Black
3	15	18	21
> 2	> 10	> 12	> 14
5	25	30	35
> 2	> 10	> 12	> 14
7	35	42	49
> 2	> 10	> 12	> 14
9	45	54	63

Pattern 45 Answers

A. 1260
If $n = 35$, then $n^2 + n = 1260$.

B. 38th
If $n^2 + n = 1482$, $n = 38$.

C. The amounts vary (see Differences table, below right). There is a constant difference when we subtract the differences. (The Differences table shows the differences between column numbers as well as the constant differences between the original differences.) The constant differences are 2, 8, 10, and 12 for the four columns. The constant differences divided by 2 are the numbers in front of n^2 in the generalizations.

D. The first $(n - 1)$ column of rods can be inverted and moved to the top of the last $(n - 1)$ column of rods to create a solid with an $n^2 + n$ or $n(n + 1)$ end section.

E. For rods of length l, the generalization is $l(n^2 + n)$, or $ln^2 + ln$.

Design Number	Number of Rods	Total Length		
		Purple	Yellow	Dark Green
1	2	8	10	12
2	6	24	30	36
3	12	48	60	72
4	20	80	100	120
5	30	120	150	180
⋮	⋮	⋮	⋮	⋮
10	110	440	550	660
⋮	⋮	⋮	⋮	⋮
20	420	1680	2100	2520
⋮	⋮	⋮	⋮	⋮
n	$n(n + 1)$ or $n^2 + n$	$4(n^2 + n)$ or $4n^2 + 4n$	$5(n^2 + n)$ or $5n^2 + 5n$	$6(n^2 + n)$ or $6n^2 + 6n$

Differences			
Number of Rods	Purple	Yellow	Dark Green
4	16	20	24
> 2	> 8	> 10	> 12
6	24	30	36
> 2	> 8	> 10	> 12
8	32	40	48
> 2	> 8	> 10	> 12
10	40	50	60

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Pattern 46 Answers

A. 1443
If $n = 38$, then $n^2 - 1 = 1443$.

B. 32nd
If $n^2 - 1 = 1023$, then $n = 32$.

C. $(n + 1)(n - 1)$: The first $(n - 2)$ columns of rods can be inverted and moved to the top of the last $(n - 2)$ columns to create a rectangular solid. The dimensions of the end section (cross section) will be $(n + 1) \times (n - 1)$.
$n^2 - 1$: The first $(n - 1)$ columns of rods can be moved to the top of the last $(n - 1)$ columns to create a rectangular solid with an end section of $n^2 - 1$.

D. The constant difference occurs the second time we subtract. (See Patterns 43–45.) This difference divided by 2 becomes the number in front of n^2 in the generalization.

Design Number	Number of Rods	Total Length		
		Light Green	Purple	Yellow
1	0	0	0	0
2	3	9	12	15
3	8	24	32	40
4	15	45	60	75
5	24	72	96	120
⋮	⋮	⋮	⋮	⋮
10	99	297	396	495
⋮	⋮	⋮	⋮	⋮
20	399	1197	1596	1995
⋮	⋮	⋮	⋮	⋮
n	$(n + 1)(n - 1)$ or $n^2 - 1$	$3(n^2 - 1)$ or $3n^2 - 3$	$4(n^2 - 1)$ or $4n^2 - 4$	$5(n^2 - 1)$ or $5n^2 - 5$

Pattern 47 Answers

A. 1122
If $n = 33$, then $n^2 + n = 1122$.

B. 36th
If $n^2 + n = 1332$, then $n = 36$.

C. You can easily take the design apart in the middle, slide the right half around 180° and push that right half up against the left half.

D. You can take the design apart in the middle; then take the right half and turn it upside down and fit it on top of the left half of the design. This will create a solid with end dimensions of $n(n + 1)$.

E. The constant differences occur the second time we subtract (see Patterns 43–45); the constant differences divided by 2 become the numbers in front of n^2 in the generalization.

Design Number	Number of Rods	Total Length		
		Dark Green	Black	Brown
1	2	12	14	16
2	6	36	42	48
3	12	72	84	96
4	20	120	140	160
5	30	180	210	240
⋮	⋮	⋮	⋮	⋮
10	110	660	770	880
⋮	⋮	⋮	⋮	⋮
20	420	2520	2940	3360
⋮	⋮	⋮	⋮	⋮
n	$(n)(n + 1)$ or $n^2 + n$	$6(n^2 + n)$ or $6n^2 + 6n$	$7(n^2 + n)$ or $7n^2 + 7n$	$8(n^2 + n)$ or $8n^2 + 8n$

Pattern 48 Answers

A. 703
If $n = 37$, then $\frac{n^2 + n}{2} = 703$.

B. 43rd
If $\frac{n^2 + n}{2} = 946$, then $n = 43$.

C. The growing number varies within each column. (The Differences table shows the differences between column numbers as well as the constant differences between the original differences.) The constant differences appear when we subtract the original differences. The constant differences are 1, 3, 4, and 5.

Design Number	Number of Rods	Total Length		
		Light Green	Purple	Yellow
1	1	3	4	5
2	3	9	12	15
3	6	18	24	30
4	10	30	40	50
5	15	45	60	75
⋮	⋮	⋮	⋮	⋮
10	55	165	220	275
⋮	⋮	⋮	⋮	⋮
20	210	630	840	1050
⋮	⋮	⋮	⋮	⋮
n	$1/2[(n)(n + 1)]$ or $\frac{n^2 + n}{2}$	$3/2[(n)(n + 1)]$ or $\frac{3(n^2 + n)}{2}$	$4/2[(n)(n + 1)]$ or $\frac{4(n^2 + n)}{2}$	$5/2[(n)(n + 1)]$ or $\frac{5(n^2 + n)}{2}$

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The constant differences divided by 2 are the numbers in front of n^2 in the generalizations.

D. For any design, taking an identical arrangement, turning it upside down, and placing it on top of the arrangement results in the end dimensions $n(n + 1)$.

	Differences		
Number of Rods	Light Green	Purple	Yellow
2	6	8	10
	> 1 > 3	> 4	> 5
3	9	12	15
	> 1 > 3	> 4	> 5
4	12	16	20
	> 1 > 3	> 4	> 5
5	15	20	25

Pattern 49 Answers

A. 2346
If $n = 34$, then $2n^2 + n = 2346$.

B. 28
If $2n^2 + n = 1596$, $n = 28$.

C. The numbers vary (see Differences table, below right). A constant difference appears when we check the second time (see Differences table). The constant differences are 4, 16, 20, and 24 for the four columns. The constant differences divided by 2 are the numbers in front of n^2 in the generalizations for each column.

D. For rods of length l, the generalization is $l(2n^2 + n)$, or $2ln^2 + ln$.

Design Number	Number of Rods	Total Length		
		Purple	Yellow	Dark Green
1	3	12	15	18
2	10	40	50	60
3	21	84	105	126
4	36	144	180	216
5	55	220	275	330
⋮	⋮	⋮	⋮	⋮
10	210	840	1050	1260
⋮	⋮	⋮	⋮	⋮
20	820	3280	4100	4920
⋮	⋮	⋮	⋮	⋮
n	$n(2n + 1)$ or $2n^2 + n$	$4(2n^2 + n)$ or $8n^2 + 4n$	$5(2n^2 + n)$ or $10n^2 + 5n$	$6(2n^2 + n)$ or $12n^2 + 6n$

	Differences		
Number of Rods	Purple	Yellow	Dark Green
7	28	35	42
	> 4 > 16	> 20	> 24
11	44	55	66
	> 4 > 16	> 20	> 24
15	60	75	90
	> 4 > 16	> 20	> 24
19	76	95	114

Pattern 50 Answers

A. 1861
If $n = 31$, then $2n^2 - 2n + 1 = 1861$.

B. 35th
If $2n^2 - 2n + 1 = 2381$, then $n = 35$.

C. The numbers grow by varying amounts (see Differences table). There is a constant difference when we check the second time. The constant differences are 4, 20, 24, and 28 for the four columns. The constant differences divided by 2 are the numbers in front of the n^2 in the generalizations for each column.

D. For rods of length l, the generalization is $l(2n^2 - 2n + 1)$, or $2ln^2 - 2ln + l$.

Design Number	Number of Rods	Total Length		
		Yellow	Dark Green	Black
1	1	5	6	7
2	5	25	30	35
3	13	65	78	91
4	25	125	150	175
5	41	205	246	287
⋮	⋮	⋮	⋮	⋮
10	181	905	1086	1267
⋮	⋮	⋮	⋮	⋮
20	761	3805	4566	5327
⋮	⋮	⋮	⋮	⋮
n	$2n^2 - 2n + 1$	$5(2n^2 - 2n + 1)$ or $10n^2 - 10n + 5$	$6(2n^2 - 2n + 1)$ or $12n^2 - 12n + 6$	$7(2n^2 - 2n + 1)$ or $14n^2 - 14n + 7$

	Differences		
Number of Rods	Yellow	Dark Green	Black
4	20	24	28
	> 4 > 20	> 24	> 28
8	40	48	56
	> 4 > 20	> 24	> 28
12	60	72	84
	> 4 > 20	> 24	> 28
16	80	96	112

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